INTERNATIONAL LIBRARY

INTERNATIONAL TEXTBOOK COMPANY
SCRANTON, PA.

Stresses in Bridge Trusses

BOOK I

By

I.C.S. STAFF

STRESSES IN BRIDGE TRUSSES
Parts 1–2

335

Published by

INTERNATIONAL TEXTBOOK COMPANY

SCRANTON, PA.

Printed in U. S. A.

INTERNATIONAL TEXTBOOK PRESS
Scranton, Pa.

89639

CONTENTS

NOTE.—This book is made up of separate parts, or sections, as indicated by their titles, and the page numbers of each usually begin with 1. In this list of contents the titles of the parts are given in the order in which they appear in the book, and under each title is a full synopsis of the subjects treated.

STRESSES IN BRIDGE TRUSSES, PART 1

STRESSES IN BRIDGE TRUSSES, PART 2

STRESSES IN BRIDGE TRUSSES
(PART 1)

Serial 943A Edition 2

INTRODUCTION

DEFINITIONS AND GENERAL CONSIDERATIONS

BEAMS AND GIRDERS

1. Wooden and Steel Beams.—The best material for a beam depends on such factors as the type of structure, the span, and the loading. Beams of short span are sometimes made of timber and are of solid rectangular cross-section, the same cross-section being used throughout the span. This is the most economical form when light beams can be used. The rectangular cross-section is not economical, however, when heavy beams are required; it must be made large enough to resist the maximum bending moment, and, as this occurs near the center of the span, some material is wasted by making the cross-section uniform, since no section between the center and the ends has to withstand so great a stress as the section at the center. For very heavy loads and long spans, it is more economical, when beams are used at all, to use steel beams. On account of the danger of fire and the frequent cost of renewal attending the employment of timber, steel beams are now almost exclusively used in permanent structures, even though their original cost may be many times that of wooden beams.

2. I Beams.—Where the bending moment is comparatively small and a low value of the section modulus is

sufficient, rolled-steel beams of uniform cross-section are used, the material in the section being so distributed as to give a comparatively large value of the section modulus for a given amount of material. Since the top and bottom of the beam are farthest from the neutral axis, as much material as possible is concentrated in these parts. The beam whose cross-section is shown in Fig. 1 fulfills this condition, and, on account of its form, is called an **I beam.** The horizontal part at the top and bottom are called the **flanges,** and the vertical part is called the **web.** Standard **I** beams are rolled in various sizes up to 24 inches in depth. Also, special **I**-shaped sections having wider flanges than the standard beams and depths up to 36 inches are manufactured at some mills. Where such beams do not provide the necessary strength to resist the bending moment, heavier built-up beams, made as explained in the next article, are used to advantage.

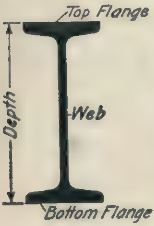


Fig. 1

3. Plate Girders.—The name **plate girder** is given to a beam, usually of steel, that has the same general form as an **I** beam, but is composed of several pieces. The cross-section of such a beam is shown in Fig. 2. The vertical part consists of a plate called the **web;** while the top and bottom parts, which consist of plates and angles, are called the **flanges.** Plate girders are generally used for spans of 25 to 100 feet, and occasionally for greater lengths.

Fig. 2

The **span** of a plate girder is the horizontal distance from center to center of its supports; the **depth** is the vertical distance between the outer surfaces of the angles that form a part of the flanges. Experience shows that the most economical plate girder has a depth of from one-seventh to one-eighth of the span. The usual practice for railroad and highway bridges is to make the depth one-eighth to one-tenth of the span, although the depth is frequently made as small as one-twelfth. These latter proportions require heavier

sections than the former, and are, therefore, very wasteful of material. For spans over 100 feet in length, it is impossible to get sufficient depth for an economical section, and the girder is so heavy that it is difficult to handle. For these reasons, plate girders are unadaptable to spans over 100 feet in length, except under special conditions.

4. In the best modern practice, the top and bottom flanges of a plate girder are made parallel throughout the entire length of the girder, and are riveted to the web. The section of the flange is decreased from the center toward the end, and so no material is wasted, as the flanges at any point are just large enough to resist the maximum bending moment that can occur at that point. In the past, plate girders were built with curved flanges. The curve was usually made a parabola, and the flange cross-section was constant from end to end. There is no additional economy in the use of girders with curved or inclined flanges. Conditions, however, sometimes require that the ends be made shallower than the center, in which case one of the flanges is inclined near the end of the girder. The web of a plate girder is very thin and requires to be stiffened at intervals with angles riveted to the sides of the web to prevent it from buckling. This subject will be more fully treated elsewhere.

5. Lattice Girder.—The **lattice girder** is sometimes used for the same span length as the plate girder, and resembles the latter in that it has flanges, usually parallel, at the top and bottom, which decrease in size from the center toward the end of the girder. Instead of the solid web, however, there is an open-web system, consisting usually of angles running diagonally from top to bottom in both directions, and riveted to vertical plates that project from between the vertical legs of the flange angles. The lattice girder belongs to a special class of structures called *trusses*, which are designed to act as beams for any span length, but are most frequently and economically used for spans over 100 feet in length. Lattice girders, however, are somewhat used for highway bridges for spans much less than 100 feet.

THE TRUSS

6. Definition.—A **truss** may be defined as a framework composed of straight pieces, called **members,** so connected as to act, to a great extent, as a rigid structure. Trusses, like beams, are designed to support loads.

The intersection of two or more members, where they are connected or joined to each other, is called a **joint.**

While the truss as a whole resists the effect of the applied forces in much the same manner as the shear and the bending moment are resisted by a solid beam, each individual member of the truss is subjected only to direct tensile or compressive stresses in the direction of its length. In order that this may be the case, the applied forces are resolved into components acting at the joints of the truss.

The simplest form of truss is a triangle, and any truss is merely an assemblage of connected triangles. As the triangle is a rigid figure whose form cannot change so long as the length of each of its sides remains the same, it is the primary and essential element of the truss.

7. Truss Members.—The upper members of a truss, such as BC, CD, etc., Fig. 3, taken together form the

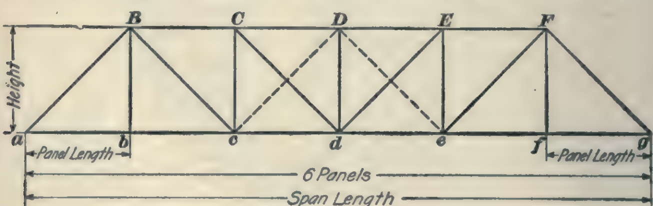


Fig. 3

upper chord of the truss; the lower members, such as ab, bc, etc., taken together form the **lower chord.** The other members of the truss, which lie between and connect the upper and lower chords, are called **web members.** The end web members aB and Fg are called **end posts.** The intermediate inclined web members, such as Bc and Cd, are called **diagonals.**

8. Panel, Span, and Height.—A **panel** is a subdivision of a truss between two consecutive joints in a chord. The joints are often called the **panel points.** The length of a panel is called the **panel length.** In Fig. 3, the points b, c, d, etc. are the joints or panel points, $a\,b$ is the panel length, and the truss is a six-panel truss. Usually, the panel lengths of a truss are equal.

9. The **span** of a truss, for purposes of stress computation, is to be taken equal to the horizontal distance between the end panel points, as shown in Fig. 3.

10. The **height,** or **depth,** of a truss, for purposes of stress computation, is to be taken equal to the vertical distance between the joints of the chords, as shown in Fig. 3.

11. The three dimensions just given—panel length, span length, and depth of truss—bear a relation to each other that, to a certain extent, decides the type of truss to be used. Engineering experience has shown that for the greatest economy the depth of truss should be about one-sixth of the length, and that the diagonal web members should make an angle with the vertical of about 40°. The panel lengths most frequently used lie between 15 and 25 feet. These values need not be strictly adhered to; there may be a reasonable departure from them without seriously increasing the cost of the truss. From the span length, the depth and panel length are so chosen as to satisfy the economical conditions as far as possible.

12. Kinds of Trusses.—A **symmetrical truss** is a truss that can be cut at the center into two parts exactly alike. If it could be folded at the center on itself in such a manner that the two ends would come together, all corresponding members in the two halves of the truss would coincide. Nearly all trusses are symmetrical.

13. A **simple truss,** like a simple beam, is one that is supported only at the ends. A **continuous truss** and a **cantilever truss** are, likewise, similar to the corresponding forms of beams.

14. Parallel-chord trusses are trusses in which the upper and lower chords are parallel. In these trusses, the panel lengths are usually equal and all the diagonal web

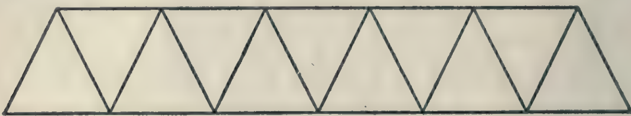


FIG. 4

members have the same inclination (see Figs. 3 and 4). The parallel-chord truss is especially adapted to short spans.

15. In **inclined-chord trusses,** which are used for long spans, the depth at the center and the panel length are so chosen that the web members near the center make an angle with the vertical of less than 40°. The panels are made the same length throughout the bridge, but the depth

FIG. 5

of the truss is decreased at each panel point from the center toward the end, inclining one or both of the chords, thereby making the web members slope differently, those near the end making an angle with the vertical of more than 40°. This saves material near the end of the truss by shortening the heavy web members, and at the same time allows an economical depth to be used at the center of the truss.

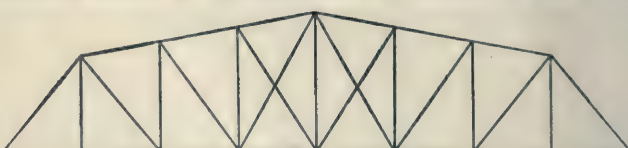


FIG. 6

When the joints of the chord are in a straight inclined line, the chord is called an **inclined chord,** and the truss, an **inclined-chord truss.** When the joints of the chord are on a curve, the chord is called a **curved chord,** and the

truss, a **curved-chord truss.** The chord is not reall curved, the members being straight between the joints. ons inclined or a curved chord gives a graceful outlir truss; on this account an inclined- or a curved-cho

FIG. 7

preferable, from an esthetic standpoint, to a truss. Examples of inclined- and curved-chorc shown in Figs. 5, 6, and 7.

which the
cted to each
an accurately
t, which is made
the ends of the
ient method of
acts, to some
ber to adjust
rge bending
practically
t its site,
structure
1 mini-
, this
road
ed

16. Multiple-System Trusses.—For long chords are sometimes made parallel and the diag members are continued across two or more panels.

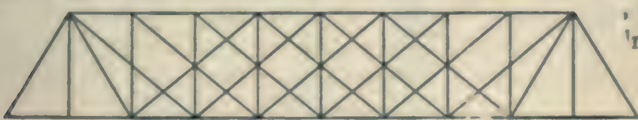


FIG. 8

of this kind are called **multiple-intersection,** or **multiple-system, trusses.** When the diagonals extend over two panels, the truss is called a **double-intersection,** or **double-system, truss;** when the diagonals extend over three or four panels, the truss is called a **triple-** or **quadruple-system truss,** and also, to some extent, a **lattice**

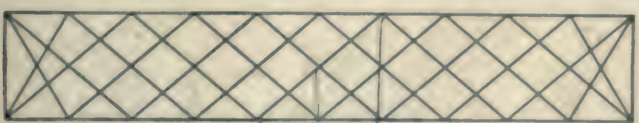


FIG. 9

truss. The arrangement has also been used in trusses with curved and inclined chords.

The multiple-system truss allows an economical choice of both the center depth and the slope of the diagonals, without making it necessary to increase the length of panel.

Trusses of this type are objectionable, however, because the up sses cannot be found directly by the ordinary conditions pane

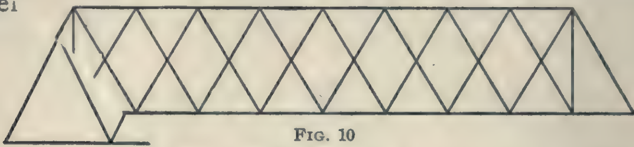


Fig. 10

Examples of multiple-system trusses are members have 8, 9, and 10.

The parallel-ch **vided-Panel Trusses.**—Another method of

15. In i omy in the depth at the center and in the slope

long spans, t
so chosen
angle with
made the

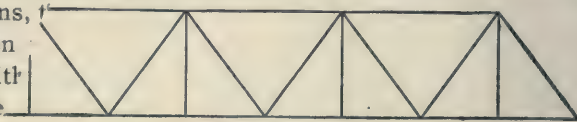


Fig. 11

diagonals, without increasing the length of the panel, subdivide the main panels of the truss by adding condary verticals, or secondary verticals and diagonals.

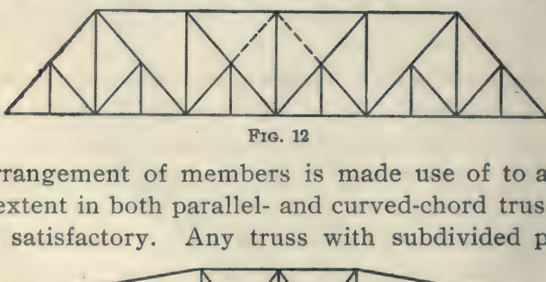


Fig. 12

This arrangement of members is made use of to a considerable extent in both parallel- and curved-chord trusses, and is very satisfactory. Any truss with subdivided panels is

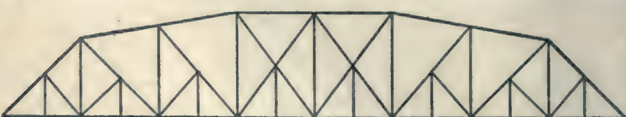


Fig. 13

called a **subdivided-panel truss.** Examples of this type are shown in Figs. 11, 12, and 13.

18. The term **girder bridge,** or **truss bridge,** is the name usually applied to a bridge simply resting on supports at the same level, so that the reactions due to vertical

loads are vertical. This distinguishes the truss bridge from the arch and the suspension bridge, in which the reactions are not vertical.

19. Pin-connected trusses are those in which the several members that meet at a joint are connected to each other, and transmit their stresses, by means of an accurately turned pin, somewhat resembling a large bolt, which is made to fit very closely into holes drilled through the ends of the members. This affords a simple and convenient method of connecting the members. As the connection acts, to some extent, like a large hinge, it allows each member to adjust itself in the line of its stress without developing large bending stresses. In this type of truss, each member is practically finished at the shop; and, in erecting the bridge at its site, the members are assembled and connected, and the structure completed, in the shortest possible time and with the minimum amount of field labor. Owing largely to this fact, this type of truss was formerly used for many highway and railroad bridges of fairly long span. At present the pin-connected truss is seldom used for spans under 500 feet.

20. Riveted trusses are those in which the several members that meet at a joint are riveted to each other, and transmit their stresses by means of connecting plates called **gussets**, to which all the members at the joint are riveted. The tendency of the best modern practice is toward such details as will give great rigidity, and for this reason the riveted truss, which is very rigid, is coming into general use for the shorter spans to which trusses are adapted. It is used for railroad bridges with spans from 100 to 500 feet and for highway bridges with spans from 50 to 500 feet.

21. Line of Action of Stress in a Truss Member. The stress in each member of a truss is considered to act in a direct line between the centers of the connections. The member itself should, therefore, be straight, and the connections at its extremities should be as nearly as possible on a line passing through the center of gravity of its cross-section. The pins in a pin-connected truss are located on lines passing

nearly through the centers of gravity of the members, and the stresses are therefore all direct stresses. There is some eccentricity in the connections of a riveted truss, due to the fact that the members are connected to large gusset plates; and, although the center lines of all the members at any joint intersect in the same point, the centers of the connections are not coincident; this causes eccentric stress.

22. Stress Sheet.—For all purposes relating to the investigation of the stresses in the members of a truss, each member is represented by a straight line indicating the line of action of its stress. A **stress sheet** of a bridge is a skeleton drawing in which the members of the bridge are shown by straight lines, which occupy positions on the drawing corresponding to the positions of the members in the bridge. On the stress sheet are shown the arrangement of floor and lateral systems, with the spacing of stringers and trusses; the span length, number of panels, panel length, and depth of truss; the lengths of diagonals; the assumed dead, live, and wind loads, and the panel loads and reactions resulting therefrom; and the maximum and minimum stresses in each member due to the assumed loads. The amount of material required in each member, and the different parts that form the cross-section of the member, are sometimes shown on the stress sheet.

TRUSS AND PLATE-GIRDER BRIDGES

MAIN PARTS

23. The main parts of a truss bridge or of a plate-girder bridge are: (1) two longitudinal vertical trusses or two-plate girders, such as have been described in the foregoing artic[illegible] (2) a *floor system;* (3) a *lateral system.*

24. The Floor System.—The traffic on a bridge is [illegible]ried by a floor, which transfers the load to the longitud[illegible] trusses or girders on the sides. In a highway bridge [illegible] floor usually consists of a slab that rests on longit[illegible]

joists, or stringers, and of cross-girders called **floor-beams,** which support the stringers and transfer the load to the trusses at the panel points. In a railroad bridge, the floor usually consists of wooden rail ties, longitudinal stringers, and floorbeams. In some truss bridges, the floor joists, or the ties, rest directly on the chord members, thus producing bending stresses in these members in addition to the direct stresses. These bending stresses must be separately computed and added to the direct stresses. This subject, however, will be left for subsequent treatment; it will here be assumed that the floor system transfers the load to the trusses at the joints, and that, therefore, the load acts at these points only. The complete analysis of a bridge includes the analysis of the stresses in the various parts of the floor system, as well as in the members of the trusses.

25. The Lateral System.—The lateral forces acting on a bridge are resisted by lateral trusses lying in the planes of the chords and connected to the latter; these trusses transmit all lateral forces to the supports. In addition to this, there are transverse braces, or frames, at each panel point, which are made as deep as the conditions will allow. The transverse brace, or frame, at the end is placed in the plane of the end posts and is usually called the **portal brace,** or simply the **portal.**

CLASSIFICATION OF BRIDGES

26. There are several ways in which bridges may be classified. One of the most common is to classify them according to the position of the floor or roadway with respect to the chords. In this classification, bridges are divided into three general types, namely: *through bridges*, *deck bridges*, and *half-through bridges.*

27. Through bridges are those that support their floors and loads at or near the level of the bottom chord, and have room for a system of lateral bracing between the top chords without interfering with the traffic. The loads pass between the trusses, or *through* the bridge. Bridges of this type

require very little space below the floor, and, therefore, the locations to which they are adapted are very common. Where trusses are of such a height that vertical transverse frames may be put in between the web members above the overhead clearance line, they are sometimes spoken of as *high-truss bridges.*

28. **Deck bridges** are those that support their floors or loads at or near the level of the upper chord. In bridges of this class, all portions of the structure are entirely below the floor. For long spans, the locations to which they are adapted are not very common. Deck bridges are more economical than any other type of bridge, and are used wherever conditions permit.

29. The chord that supports the floor system is called the **loaded chord;** the other, the **unloaded chord.** In a through bridge, the loaded chord is the lower chord; in a deck bridge, the loaded chord is the upper chord.

30. **Half-through bridges** are those that support their floors or loads at some elevation intermediate between the top and the bottom chord, or at the bottom chord, and the trusses of which are not deep enough to allow a system of overhead bracing. When plate girders are used, such a bridge is called a half-through plate-girder bridge. When trusses are used, such a bridge is called a **low-truss, or pony-truss, bridge.**

31. Bridges are also spoken of as *plate-girder bridges*, *riveted-truss bridges*, and *pin-connected truss bridges*, according to the type of beam or truss that supports the loads. Bridges may be further classified according to the style of truss, giving two general classes; namely, **parallel-chord bridges** and **inclined-chord bridges.** Each of these classes may be subdivided into single-system, multiple-system, and subdivided-panel bridges, as explained in connection with trusses

LOADS AND REACTIONS

CLASSIFICATION OF LOADS

32. External Forces Acting on a Bridge.—The external forces acting on a bridge consist of: (1) the weight of the structure itself; (2) the weight of whatever the structure is designed to support; (3) the pressure of the wind; and (4) the reactions of the abutments. The first two of these classes of forces are called **loads**. In addition to the forces just mentioned, it is sometimes necessary to consider other applied forces, such as the centrifugal force of a train moving on a curved track, and the horizontal force caused by moving trains.

33. Kinds of Loads.—Loads are divided into two general classes, namely: *dead loads* and *live loads*.

The **dead load** consists of the weight of the structure itself, including the track or floor, the floor system, and the girders or trusses. It is the force of gravity acting on every part of the structure, and is, therefore, actually applied at all points. The weight of the floor system is transferred to the joints of the loaded chord by the floor-beams. The weight of the truss is transferred to the joints of the loaded and unloaded chords by the members themselves. Methods of estimating in advance the approximate weight of floor systems and trusses will be given elsewhere. The dead load is usually assumed to be a uniform amount per linear foot of structure. For short spans, up to about 125 feet, the direct stresses due to dead load are found by assuming all the load to be applied at the joints of the loaded chord. For longer spans, a part of the load, usually one-third, is assumed to act at the joints of the unloaded chord. This assumption leads to results that are very close to the actual stresses.

34. The **live load** is the load due to the traffic. It is therefore a **moving** load, and is often so called. For highway bridges, the live load is assumed to be a specified amount per square foot, uniformly distributed over the roadway. For bridges that are subject to the passage of heavy loads concentrated on wheels, the live load is assumed to be a certain arrangement of concentrated loads, or wheel loads, which may be taken by themselves or in connection with the uniform load. For railroad bridges, the live load usually consists of a system of concentrated wheel loads representing a type of locomotive, or of certain uniform loads that will give an approximately equivalent effect. The calculation of stresses due to uniform and to concentrated loads require separate consideration. This subject will be fully treated elsewhere. The amount of live load that a bridge is designed to carry is sometimes called the **capacity** of the bridge.

35. Wind Pressure.—The wind pressure, sometimes called the **wind load**, is the force exerted by the wind on all surfaces exposed to it. These surfaces consist of the sides of the members and the exposed surfaces of the moving loads that cross the structure. In highway bridges, the surface of the loads is usually very small compared to the exposed surface of the bridge. In railroad bridges, it is necessary to consider the pressure of the wind against the side of a train, in addition to the pressure against the exposed surface of the structure. It is customary to assume the wind pressure as a uniformly distributed force, and express it in pounds per square foot of exposed surface, or in pounds per linear foot of the loaded or unloaded chord.

36. Centrifugal Force.—The centrifugal force considered in bridges is the pressure exerted by a train of cars moving on a curved track; it acts radially outwards, and is transferred by the floor system to the joints of the loaded chord. It is usually expressed as a percentage of the live load and depends on the degree of curvature of the track, ard on the weight and speed of the train.

37. The **longitudinal thrust** is the force exerted on a structure by a train of cars crossing the structure while the brakes are set. This force is a maximum when the brakes are set hard enough to prevent the wheels from turning, in which case they slide on the rails.

The **tractive force** is the force exerted on a structure by the friction of the driving wheels of a locomotive drawing a train of cars; it is usually less than the longitudinal thrust, and may be neglected if the latter is taken into consideration.

PANEL LOADS

38. Definition.—The amount of load that is transferred to a joint of the loaded chord is called **a panel load,** or **panel concentration.**

39. Dead Panel Loads.—As the weight of the floor system is a large part of the dead load, the dead panel loads of the loaded chord are larger than those of the unloaded chord. If all the dead load were considered as applied along the loaded chord, each panel load would equal the uniform load per linear foot multiplied by the length of one panel and divided by 2 (there being two trusses to the bridge). It is convenient to compute first this panel load, and then, if considered necessary, assume an approximate distribution of it between the loaded and the unloaded chord, as explained in Art. 42.

40. Live Panel Loads.—All the live load is applied along the loaded chord. In highway bridges, the load per linear foot is equal to the specified load per square foot on the floor, multiplied by the clear width of the roadway, plus a like product for the sidewalks, if any. The panel load is then equal to one-half this total uniform load multiplied by a panel length. Some highway bridges have only one side-walk, which is sometimes supported outside of the truss. In this case, the load on and stresses in each truss must be computed separately. In railway bridges, if the load is a uniform load per linear foot, the panel load is found as just

stated; if it consists of a series of wheel loads, the panel loads will vary in amount. The treatment of this class of loads is considered in a subsequent Section.

41. Wind Panel Load.—The wind pressure per linear foot, assumed or computed for either chord, multiplied by the panel length gives the wind panel load for that chord.

42. Illustrative Example.—As an example, let a through highway bridge, such as is shown in Fig. 14, contain seven equal panels, each 20 feet long, and have a clear width of roadway of 16 feet. Suppose the dead load to be 600 pounds per linear foot, and the live load to be 100 pounds

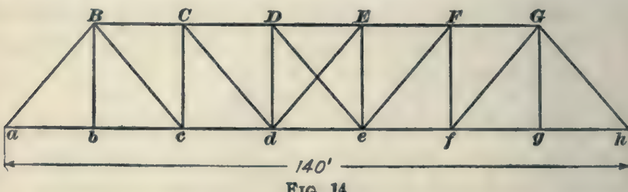


FIG. 14

per square foot of roadway. The dead panel load will then be

$$\frac{600 \times 20}{2} = 6{,}000 \text{ pounds}$$

and the live panel load,

$$\frac{100 \times 16 \times 20}{2} = 16{,}000 \text{ pounds}$$

Assuming all the dead load to be applied at the joints of the loaded chord, each of the joints $b, c, d, e, f,$ and g will be loaded with 6,000 pounds dead load; while, if the bridge sustains a live load throughout its length, each of the joints will also have a load of 16,000 pounds. At the points a and h, there is a half-panel load consisting of 3,000 pounds dead load and 8,000 pounds live load. These half-panel loads are carried directly by the supports and do not affect the trusses, and hence can be omitted in the calculation of stresses. A seven-panel truss would thus be considered as loaded with six intermediate panel loads; a six-panel truss would likewise have five intermediate panel loads; etc. If it is desired to distribute the dead load between the lower and

the upper chord, one-third of the load, or 2,000 pounds, may be assumed as acting at each of the upper joints, and 4,000 pounds at each of the lower joints, except at *a* and *h*, each of which would still carry the half-panel load of 3,000 pounds.

EXAMPLES FOR PRACTICE

1. A bridge 99 feet long is designed to sustain a live load of 100 pounds per square foot on a roadway 16 feet wide, clear width. The trusses are divided into 6 panels. What is: (*a*) the live load per linear foot? (*b*) the panel live load?

Ans. $\begin{cases} (a) & 1{,}600 \text{ lb.} \\ (b) & 13{,}200 \text{ lb.} \end{cases}$

2. An eight-panel bridge of 120 feet span carries a roadway 18 feet wide, clear width. The live load assumed for the trusses is 96 pounds per square foot of roadway. What is: (*a*) the live load per linear foot? (*b*) the panel live load?

Ans. $\begin{cases} (a) & 1{,}728 \text{ lb.} \\ (b) & 12{,}960 \text{ lb.} \end{cases}$

3. If for the bridge of example 2 the wind load per linear foot is assumed as 300 pounds for the lower chord and 150 pounds for the upper chord, what is the panel wind load: (*a*) for the lower chord? (*b*) for the upper chord?

Ans. $\begin{cases} (a) & 4{,}500 \text{ lb.} \\ (b) & 2{,}250 \text{ lb.} \end{cases}$

4. Suppose, for the bridge described in example 2, that the dead load is assumed to be 760 pounds per linear foot. What will the panel dead load be? Ans. 5,700 lb.

REACTIONS

43. In all bridge trusses, one end is free to move horizontally in a longitudinal direction, so that the reactions due to the dead and live loads will be vertical. The effect of wind pressure and other horizontal forces will receive separate consideration.

44. Dead-Load Reactions.—By the principles of statics, the reactions exerted by the supports must hold in equilibrium the applied loads. Let Fig. 15 represent any truss acted on by the panel loads W as shown. Let R_1 and R_2 be the reactions. The loads and reactions together comprise all the external forces acting on the structure, and any of the conditions of equilibrium may be applied to these forces. The object being to determine the values of the reactions R_1 and R_2, it will be convenient to take moments

of the forces about the point f. The following equation therefore obtains:

$$\Sigma M_f = R_1 \times 100 - W \times 80 - W \times 60 - W \times 40 - W \times 20 = 0$$

whence
$$R_1 = \frac{W \times 200}{100} = 2\,W$$

In like manner, by taking moments about a, the reaction R_2

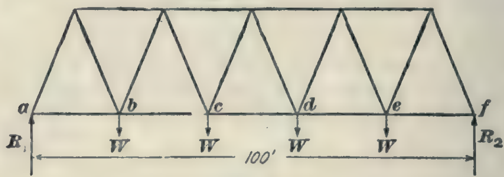


FIG. 15

is found to be $2\,W$. Or, by putting the sum of the vertical forces equal to zero,

$$R_1 - 4\,W + R_2 = 0;$$

whence, as before,

$$R_2 = 4\,W - R_1 = 4\,W - 2\,W = 2\,W$$

In this case, where the load is symmetrical, it is evidently unnecessary to write out these equations, since the reactions must be equal, and each must be equal to one-half the total load, or to $2\,W$.

45. Live-Load Reactions.—If the truss is only partly loaded, as in Fig. 16, the reactions are not equal, and it is necessary to calculate them by applying the principles of

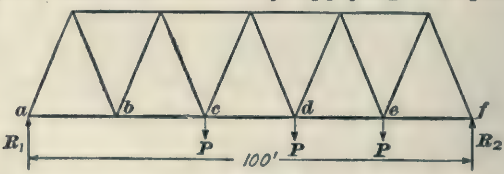


FIG. 16

statics as just stated. In the truss shown, with f as a center of moments, we have

$$\Sigma M_f = R_1 \times 100 - P \times 60 - P \times 40 - P \times 20 = 0; \qquad (1)$$

whence
$$R_1 = \frac{P \times 120}{100} = 1.2\,P$$

Also,
$$R_1 + R_2 - 3\,P = 0;$$

whence
$$R_2 = 3\,P - R_1 = 3\,P - 1.2\,P = 1.8\,P$$

This method of calculation is perfectly general and applies as well when the panel loads or panel lengths are unequal.

46. Where the panel lengths are equal, as is usually the case, the reactions for a partial load, as in Fig. 16, can be found more readily by writing equation (1), Art. **45**, in a little different form. Taking the panel length as the unit of length, and changing the order of arrangement, we have

$$R_1 \times 5 - P \times 1 - P \times 2 - P \times 3 = 0;$$

whence
$$R_1 \times 5 = 1\,P + 2\,P + 3\,P$$

and
$$R_1 = \tfrac{1}{5}\,P + \tfrac{2}{5}\,P + \tfrac{3}{5}\,P$$

Thus, it is seen that the left reaction is equal to one-fifth the load at *e*, plus two-fifths the load at *d*, plus three-fifths the load at *c*. This statement can be written at once by inspection: for, as regards the load at *e*, it is known, from the principle of moments, that one-fifth will be carried by the left support, and four-fifths by the right; likewise, two-fifths of the load at *d* will be carried at *a* and three-fifths at *f*, etc. This method of getting reactions for partial loads, *when the panel lengths are equal*, is very convenient and will be frequently employed hereafter.

EXAMPLE.—(*a*) Assuming the truss shown in Fig. 17 to be loaded, at each of the lower chord joints with a load of 5,000 pounds, to calculate the reactions. (*b*) Assuming the joints *d*, *e*, and *f* only, to be loaded with a load of 5,000 pounds each, to calculate the reactions.

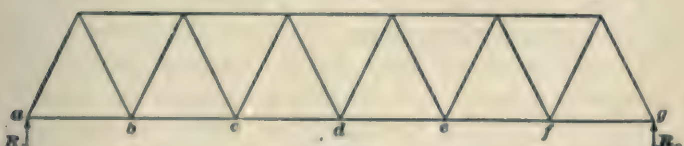


FIG. 17

SOLUTION.—(*a*) In this case, the load is symmetrical; therefore,

$$R_1 = R_2 = \frac{5 \times 5{,}000}{2} = 12{,}500 \text{ lb.} \quad \text{Ans.}$$

(*b*) Taking moments about *g*,

$$R_1 \times 6 - 5{,}000 \times 1 - 5{,}000 \times 2 - 5{,}000 \times 3 = 0;$$

whence

$$R_1 = \frac{5{,}000 \times 1 + 5{,}000 \times 2 + 5{,}000 \times 3}{6} = 5{,}000 \text{ lb.} \quad \text{Ans.}$$

Taking moments about a

$$-R_2 \times 6 + 5{,}000 \times 3 + 5{,}000 \times 4 + 5{,}000 \times 5 = 0;$$

whence

$$R_2 = \frac{5{,}000 \times 3 + 5{,}000 \times 4 + 5{,}000 \times 5}{6} = 10{,}000 \text{ lb.} \quad \textbf{Ans.}$$

EXAMPLES FOR PRACTICE

1. If the truss shown in Fig. 17 carries a panel load of 6,000 pounds at each of the lower chord joints, what are the two reactions?

$$\text{Ans.} \begin{cases} R_1 = 15{,}000 \text{ lb.} \\ R_2 = 15{,}000 \text{ lb.} \end{cases}$$

2. If the truss shown in Fig. 17 carries a panel load of 6,000 pounds at each of the joints b, c, **and** d, **what are the two reactions?**

$$\text{Ans.} \begin{cases} R_1 = 12{,}000 \text{ lb.} \\ R_2 = 6{,}000 \text{ lb.} \end{cases}$$

SHEARS AND MOMENTS

MAXIMUM MOMENTS AND SHEARS IN BEAMS

BENDING MOMENTS

47. It is desirable here to review some of the principles previously explained in connection with shears and moments in beams, and to add sufficient further discussion to make complete the analysis of maximum moments and shears in simple beams and trusses for fixed and for moving uniform loads. In designing a beam, it is necessary to know the maximum bending moments and the maximum shears at various sections. From the former, the flange or fiber stresses are found; and from the latter, the shearing or web stresses. It will be sufficient here to deal with this subject in a general manner, stating the fundamental principles. The application of these principles to the calculation of the actual flange and web stresses will be taken up in connection with design, where it can be more conveniently treated.

48. Bending Moments Due to a Uniform Dead Load. To calculate the bending moment at any section of a beam, it is merely necessary to determine the algebraic sum of the

moments of all the external forces to the left or right of the section about the neutral axis of the section. In Fig. 18 (*a*), AB is a beam of length l, loaded with a uniform load of

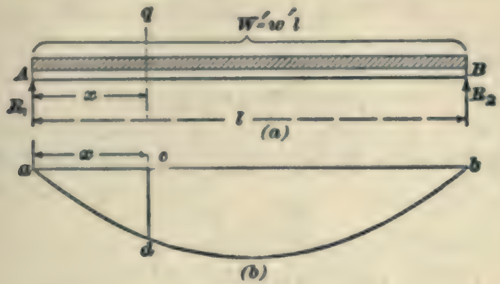


FIG. 18

w' units of weight per unit of length. The total load is $w'l$, and each reaction is equal to $\frac{w'l}{2}$. The bending moment M' at any section q, distant x from the left end, is given by the formula

$$M' = R_1 x - w' x \times \frac{x}{2};$$

or, substituting the value of R_1, and factoring,

$$M' = \frac{w'}{2}(l - x)x \qquad (1)$$

This moment is a maximum at the center, where its value is given by the formula

$$\text{Max. } M' = \frac{w'l^2}{8} \qquad (2)$$

The bending moments at all points of the beam due to this uniform load are represented graphically in Fig. 18 (*b*) by the moment curve adb, which is a parabola and shows how the moment varies along the beam.

49. Bending Moment Due to a Uniform Live Load. The maximum bending moment at any section of a simple beam, caused by a uniform live load, will occur when the load extends entirely across the span. The beam being fully loaded for maximum moments, these moments will be given by formula 1 of the preceding article, except that w'

should be replaced by the live load w'' per unit of length. If, therefore, the maximum live-load moment at the point q is denoted by M'', then

$$M'' = \frac{w''}{2}(l - x)\,x \qquad (1)$$

From the preceding article,

$$M' = \frac{w'}{2}(l - x)\,x$$

Dividing the first by the second of these two equations, the result is

$$\frac{M''}{M'} = \frac{w''}{w'};$$

whence $$M'' = M' \times \frac{w''}{w'} \qquad (2)$$

When, therefore, the dead-load moment at any section of the beam has been determined, the maximum live-load moment at the same section is obtained by multiplying the dead-load moment by the ratio $\frac{w''}{w'}$ of the live load per unit of length to the dead load per unit of length.

If the total or resultant bending moment at q is denoted by M, then

$$M = M' + M'';$$

or, substituting the values of M' and M'',

$$M = \frac{w' + w''}{2}(l - x)\,x \qquad (3)$$

EXAMPLE.—A beam 40 feet long supports a dead load of 500 pounds per foot and a live load of 1,800 pounds per foot. What are the dead-load and maximum live-load bending moments at points 10 feet apart along the beam?

SOLUTION.—The moments will be found by means of formula **1**, Art. **48**, and formula **2**, Art. **49**. Here, $l = 40$, $w' = 500$ lb., $w'' = 1{,}800$ lb., and

$$\frac{w''}{w'} = \frac{1{,}800}{500} = 3.6$$

The values of x for the several points are 0, 10, 20, 30, and 40. The moments, in foot-pounds, are as follows:

For $x = 0$, $M' = \frac{500}{2}(40 - 0)\,0 = 0$, $M'' = 0$.

For $x = 10$, $M' = 250\,(40 - 10)\,10 = 75{,}000$, $M'' = 75{,}000 \times 3.6 = 270{,}000$.

For $x = 20$, $M' = 250(40 - 20)20 = 100,000$, $M'' = 100,000 \times 3.6 = 360,000$.

For $x = 30$, $M' = 250(40 - 30)30 = 75,000$, $M'' = 75,000 \times 3.6 = 270,000$.

For $x = 40$, $M' = 250(40 - 40)40 = 0$, $M'' = 0$.

EXAMPLES FOR PRACTICE

1. A beam 100 feet long supports a dead load of 600 pounds per foot. What is the dead-load moment: (*a*) at the center? (*b*) at a point 75 feet from the left end?
Ans. (*a*) 750,000 ft.-lb. (*b*) 562,500 ft.-lb.

2. A beam 40 feet long supports a live load of 750 pounds per foot. What is the live-load moment at a point: (*a*) 10 feet from the left support? (*b*) 20 feet? (*c*) 30 feet?
Ans. (*a*) 112,500 ft.-lb. (*b*) 150,000 ft.-lb. (*c*) 112,500 ft.-lb.

3. A beam 80 feet long supports a dead load of 450 pounds per foot and a live load of 1,200 pounds per foot. What is: (*a*) the dead-load bending moment, and (*b*) the live-load bending moment at a point 30 feet from the left support?
Ans. (*a*) 337,500 ft.-lb. (*b*) 900,000 ft.-lb.

4. In example 3, what is the total bending moment at the center of the beam? Ans. 1,320,000 ft.-lb.

SHEARS

50. Shear and Its Sign.—The shear at any section of a beam is defined as the sum of all the vertical forces acting on the beam to the left or right of the section. The correct

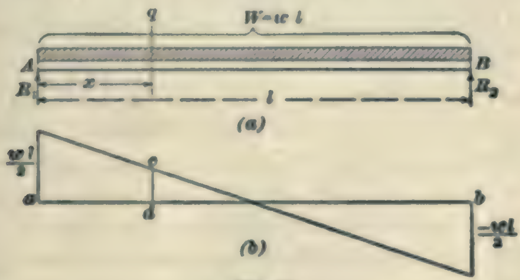


FIG. 19

sign will be given by considering the forces on the left and taking upward forces as positive and downward forces as

negative. If the load is a uniform load w per unit of length Fig. 19 (a), the shear at any point whose distance from the left end is x is given by the formula

$$V = R_1 - w\,x = \frac{w\,l}{2} - w\,x$$

At the left end of the beam, where $x = 0$, the shear is equal to R_1, or $\frac{w\,l}{2}$; at the center, where $x = \frac{l}{2}$, the shear is 0; and at the right end, where $x = l$, the shear is $-\frac{w\,l}{2}$, or $-R_2$. The shear diagram for dead load is shown in Fig. 19 (b).

It is to be noted that the shears on the right of the center are negative, and those on the left are positive, but that for two points equidistant from the center the shears are of the same numerical value.

51. The meaning of positive and negative shears will be made clearer by a study of Fig. 20, which shows how the forces acting tend to shear the beam at any given section. *Positive shear* at any section q occurs when the external forces tend to move the left-hand portion upwards and the right-hand portion downwards; *negative shear* occurs when the external forces tend to move the left-hand portion downwards and the right-hand portion upwards.

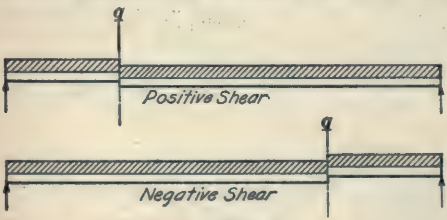


FIG. 20

52. Live-Load Shears.—In order to arrive at a general rule for determining the maximum live-load shear at any section of a beam, it will be convenient to consider first the effect of a single load W, Fig. 21, placed anywhere on the beam. Let q be the section at which the shear is required. Considering the weight W first at any point on the *right* of the section, it is seen that the only external force acting on the left of the section will be the reaction. The shear at the section will then be equal to this reaction, and will be *positive.*

If the load W is placed anywhere to the *left* of the section, the shear will be equal to the left reaction *minus* the load W; and, as the reaction is always less than the load, the resultant of the reaction and the load will act *downwards*, and the shear will be *negative*.

It is thus seen that a load placed anywhere on the right of a section will cause positive shear in that section, and if placed

FIG. 21

anywhere on the left of the section, it will cause negative shear. From this it follows that, if a number of loads is to be placed on a beam so as to cause the maximum positive shear at a given section, as many loads as possible should be placed on the right of the section, and none on the left. For maximum negative shear, the reverse would be true. If the given load is a uniform live load, the maximum positive shear at any section obtains when the beam is loaded on the right of the section only, and the maximum negative shear, when the beam is loaded on the left.

53. Applying the rule just given to any beam AB, Fig. 22 (a), the maximum positive live-load shear at any

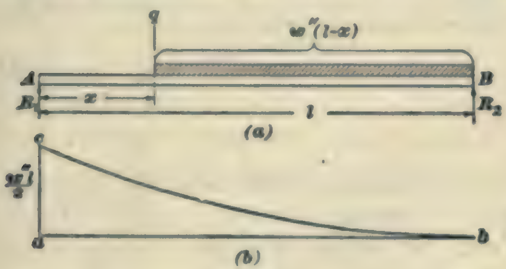


FIG. 22

section q, distant x from the left end, due to a uniform load of w'' per unit of length, obtains when the load covers all the part of the beam to the right of q. The shear, which will be denoted by V'', will be equal to the left reaction. Taking

moments about B, and noting that the total load on the beam is equal to $w''(l-x)$, and that its lever arm about B is equal to $\frac{l-x}{2}$, the following equation obtains:

$$R_1 \times l - w''(l-x) \times \frac{l-x}{2} = 0;$$

whence $$R_1 = \frac{w''(l-x)\left(\frac{l-x}{2}\right)}{l} = \frac{w''}{2l}(l-x)^2$$

As the maximum live-load shear is equal to R_1, then

$$V'' = \frac{w''}{2l}(l-x)^2$$

If the load covers the whole beam, $x = 0$, and

$$V'' = \frac{w''l^2}{2l} = \frac{w''l}{2}$$

If the load covers half the beam, $x = \frac{l}{2}$, and, therefore,

$$V'' = \frac{w''}{2l}\left(l - \frac{l}{2}\right)^2 = \frac{w''}{2l}\left(\frac{l}{2}\right)^2 = \frac{w''}{2l} \times \frac{l^2}{4} = \frac{w''l}{8}$$

In Fig. 22 (b), the ordinates to the curve cb represent the

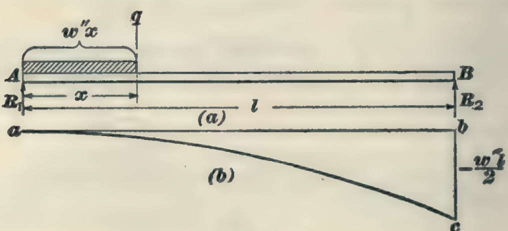


FIG. 23

maximum positive shears in the beam at all points, obtained by giving to x values between l and 0. This curve is a parabola with its vertex at b, and may be constructed by various methods described elsewhere.

54. The maximum live-load negative shear at any point in the beam will occur when the beam is loaded on the left of that point, as shown in Fig. 23 (a). The maximum shear at section q will equal the left reaction minus the load $w''x$

The left reaction R_1 is found by taking moments, as before, about the point B:

$$R_1 = \frac{w'' x\left(l - \frac{x}{2}\right)}{l} = w'' x - \frac{w'' x^2}{2l}$$

and the maximum negative shear is

$$V'' = R_1 - w'' x = -\frac{w'' x^2}{2l}$$

This value is equal to the right reaction, and might have been found by taking moments about A, without previously computing R_1. Fig. 23 (b) shows the shear diagram for negative shears.

By examining the values of the negative shears, beginning at the right end and passing toward the left, it will be observed that the maximum positive shear at any point is numerically equal to the maximum negative shear at a point in the opposite end of the beam equidistant from the center.

55. Illustrative Example.—Let it be required to determine the dead-load shears at sections 5 feet apart in a beam 60 feet long, Fig. 24. Let the dead load be 400 pounds

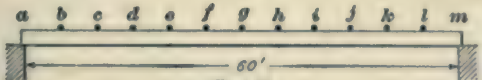


FIG. 24

($= w'$) per linear foot, and the live load 1,200 pounds ($= w''$) per linear foot.

1. *Dead-Load Shears.*—The reactions are each equal to

$$\frac{400 \times 60}{2} = 12{,}000 \text{ pounds}$$

The shears will be found by the formula of Art. **50**, substituting w', or 400, for w.

At a, $x = 0$, and $V' = R_1 = +12{,}000$ pounds.

At b, $x = 5$, and $V' = 12{,}000 - 400 \times 5 = +10{,}000$ pounds.

At c, $x = 10$, and $V' = 12{,}000 - 400 \times 10 = +8{,}000$ pounds.

In like manner, the shear for each of the other given sections is found. The results are given in the second column of the accompanying table.

TABLE OF SHEARS

1	2	3	4	5	6
Section	Dead-Load Shear	Maximum Live-Load Shear		Combined Shear	
		Positive	Negative	Dead-Load and Maximum Positive Live-Load Shear	Dead-Load and Maximum Negative Live-Load Shear
a	+12,000	+36,000	0	+48,000	+12,000
b	+10,000	+30,250	− 250	+40,250	+ 9,750
c	+ 8,000	+25,000	− 1,000	+33,000	+ 7,000
d	+ 6,000	+20,250	− 2,250	+26,250	+ 3,750
e	+ 4,000	+16,000	− 4,000	+20,000	0
f	+ 2,000	+12,250	− 6,250	+14,250	− 4,250
g(center)	0	+ 9,000	− 9,000	+ 9,000	− 9,000
h	− 2,000	+ 6,250	−12,250	+ 4,250	−14,250
i	− 4,000	+ 4,000	−16,000	0	−20,000
j	− 6,000	+ 2,250	−20,250	− 3,750	−26,250
k	− 8,000	+ 1,000	−25,000	− 7,000	−33,000
l	−10,000	+ 250	−30,250	− 9,750	−40,250
m	−12,000	0	−36,000	−12,000	−48,000

It is to be noted that the shears beyond *g*, the center section, may be written out at once from the values to the left of this section.

2. *Live-Load Shears.*—The maximum positive live-load shears are obtained from the formula of Art. **53.** At *a*, where $x = 0$,

$$V'' = R_1 = \frac{1{,}200 \times 60}{2} = +\ 36{,}000 \text{ pounds}$$

At *b*, $x = 5$, and

$$V'' = \frac{1{,}200}{2 \times 60}(60 - 5)^2 = +\ 30{,}250 \text{ pounds}$$

At *c*, $x = 10$, and

$$V'' = \frac{1{,}200}{2 \times 60}(60 - 10)^2 = +\ 25{,}000 \text{ pounds}$$

and so on. These shears are given in the third column of the above table. The maximum negative live-load shears are found in like manner, by the use of the formula in Art. 54. The fifth and the sixth column of the table contain the combined shears, found as explained below.

3. *Combined Shears.*—Since the dead load is always present, the maximum total shear at any section of the beam will be found by combining the dead-load snear with one or the other of the maximum live-load shears. Combining with the maximum positive shears, the fifth column of the preceding table is obtained; while the sixth column is obtained by combining the dead-load shears with the maximum negative live-load shears. By comparing these two columns, it will be noticed that, for sections to the left of the center, the maximum shears are the positive shears of column 5; while to the right of the center they are the negative shears of column 6. For two sections equidistant from the center, the maximum shears are numerically equal.

It is to be further noted that in column 5 there is a small positive shear at the sections *h*, zero shear at *i*, and negative shears beyond. These negative shears, which are smaller than the dead-load shears, are equal to the difference between the dead-load negative and the live-load positive shears. They are in fact the least negative shears that can occur in the beam. In the same way, column 6 gives, in the upper part, the least positive shears down as far as *e*, then the maximum negative shears for the remainder of the beam. Taking the two columns together, the greatest *range* of shear is obtained that can possibly occur in the beam at the several sections for any positions of the live load. Thus, at section *a*, the shear may be as high as + 48,000 and as low as + 12,000; at *b*, the limits are + 40.250 and + 9,750; at *e*, + 20,000 and 0; at *f*, + 14,250 and − 4,250; and at *g*, + 9,000 and − 9,000. On the right end, the limits are the same numerically at corresponding sections, but are of opposite sign. From this discussion, it is plain that only positive shears can exist from *a* to *e*; both kinds are possible from *e* to *i*; and only negative shears beyond *i*. This table should

be carefully studied, as the relations here illustrated are of great importance in the analysis of trusses.

4. *Summary of Results.*—Summarizing the results of the foregoing analysis, the following statement may be made regarding a beam subjected to both a dead and a uniform live load: (1) The *maximum shear* at any section to the left of the center will be *positive*, and may be found by adding to the dead-load shear the maximum positive live-load shear, the beam being loaded to the *right* of the section. (2) The *minimum shear* at any section to the left of the center may be found by combining the dead-load shear with the maximum negative live-load shear, the beam being loaded to the *left* of the section. Near the end of the beam, these minimum shears will be positive, but near the center, where the live-load negative shears exceed numerically the dead-load positive shears, the resulting values will be negative. For sections on the right of the center, exactly the reverse of this holds true: the shears are numerically equal to, but of opposite sign from, those on the left of the center.

56. In the design of beams and plate girders, the sign of the shear is usually immaterial, and the maximum numerical value is all that is needed. In the case of trusses, however, it is essential to know the sign of the shear and of the stresses resulting therefrom. Furthermore, in many cases it will be necessary to find not only the maximum shear and stress but also the minimum values, so that the greatest range of stress to which the member is subjected may be known.

MAXIMUM MOMENTS AND SHEARS IN TRUSSES

57. Notation—Reactions.—So far, it has been assumed that the load was applied at every point along the beam. In the case of trusses and plate-girder bridges with floor systems, the load is applied along the stringers and transmitted to the trusses or girders by the floorbeams. A truss, for example, is subjected to loads acting at the joints of the chords. It is necessary to find the maximum moments

at the joints and the maximum shears in the panels. In what follows, l will denote the length of the span; p, the length of one panel; n, the number of panels; w', the dead load per unit of length; w'', the live load per unit of length; W', the panel dead load; W'', the panel live load; R_1, the left reaction; and R_2, the right reaction. When necessary to distinguish between reactions due to the dead and to the live load, the former will be denoted by one accent—R_1', R_2'; the latter, by two—R_1'', R_2''.

58. The loads W', W'' are taken as acting at the joints. By referring to Fig. 25, where $n = 6$, it will be observed that the number of panel points (the end joints are not counted)

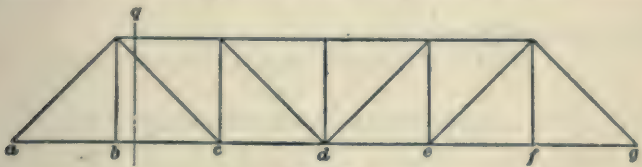


FIG. 25

is 5, or $6 - 1$. In general, the number of loaded joints is $n - 1$, or one less than the number of panels. This being the case, the total dead load on the truss is $(n - 1)\,W'$, and the total live load, when all the truss is loaded, is $(n - 1)\,W''$. Therefore, in this case,

$$R_1' = \tfrac{1}{2}(n - 1)\,W' = \tfrac{1}{2}(n - 1)\,p\,w'$$
$$R_1'' = \tfrac{1}{2}(n - 1)\,W'' = \tfrac{1}{2}(n - 1)\,p\,w''$$

It should be kept in mind that w' and w'' are, in this discussion, loads per unit of length for *one truss*, not for the *bridge*.

59. Dead-Load Moments and Shears.—The dead-load moments and shears are determined as for a beam loaded at various points. In the case of a truss, those points are the panel points, each of which carries a load equal to W'. Thus, the moment at b, Fig. 25, is $R_1'\,p$; at c, $R_1' \times 2\,p - W'\,p$; at d, $R_1' \times 3\,p - W' \times 2\,p - W'\,p$; etc. The shear between a and b is R_1'; between b and c, $R_1' - W'$; between c and d, $R_1' - 2\,W'$; etc. The shear between a and b

is referred to as the shear in the panel ab; that between b and c, as the shear in the panel bc; etc.

EXAMPLE.—A truss bridge whose span is 120 feet carries a dead load of 800 pounds per linear foot. If each truss is divided into six panels, as indicated in Fig. 25, what is the dead-load shear for one truss in the second panel from the left end, as panel bc?

SOLUTION.—The panel length is $120 \div 6 = 20$ ft., and the panel dead load for one truss is

$$W' = \frac{800 \times 20}{2} = 8{,}000 \text{ lb.}$$

Since there are five intermediate panel loads on the truss and either dead-load reaction is equal to half of the total load,

$$R_1' = \frac{5 \times 8{,}000}{2} = 20{,}000 \text{ lb.}$$

The required shear in panel bc is

$$V' = R_1' - W' = 20{,}000 - 8{,}000 = 12{,}000 \text{ lb.} \quad \text{Ans.}$$

60. Live-Load Moments.—The maximum live-load bending moment at any joint occurs when the whole truss is loaded; that is, when a load equal to W'' is applied at every joint of the loaded chord; and is calculated in the same manner as the dead-load moment. Thus, the maximum live-load bending moment at d is equal to

$$R_1'' \times 3p - W'' \times 2p - W'' \times p$$

61. Exact Live-Load Shears.—For the maximum positive live-load shear at any section in a beam, it has been shown that the beam should be fully loaded up to that section from the right, and should have no load on the left of the section. In a truss that is loaded only at the panel points, as is usually the case in bridge trusses, the shear is the same at all points in a panel. Thus, the shear in the panel bc of the truss shown in Fig. 25 is the shear on any section q in that panel. This shear is equal to the difference between the left reaction and the load—if there is one—at the joint b. For a maximum positive shear in panel bc, the live load should extend up to joint c at least, for up to this point the reaction is the only force acting on the left of the section, and this increases as the load is moved from the right until it reaches c. If the load advances beyond c, the

joint b will begin to receive some load, and the shear will then be equal to the left reaction minus the load at b. As the load continues to move to the left, the reaction is increased, but the load at b is also increased, so that the shear in the panel may be increased or decreased by such movement, according to the relative increase in the left reaction and the load at b. There is a certain point in each panel to which the load should extend in order that the shear in that panel may be a maximum. The condition of loading for the maximum positive shear in the second panel of a six-panel truss is represented in Fig. 26. The

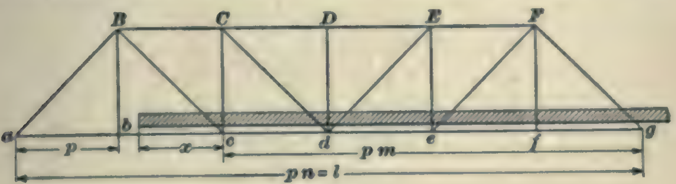


FIG. 26

distance x from the joint c to the end of the load can be determined exactly, as it depends only on the panel length p; the number of panels m between the joint c and the right-hand end of the span, or the number of panels that are fully loaded; and the number of panels n in the truss. Thus,

$$x = \frac{pm}{n-1} \qquad (1)$$

However, the distance x need not be calculated in order to find the maximum shear, as the shear can be computed directly by the following formula:

$$V'' = w''p \times \frac{m^2}{2(n-1)} \qquad (2)$$

in which V'' = maximum live-load shear in any panel of a truss, in pounds;

w'' = amount of uniform load per linear foot for one truss, in pounds;

p = panel length, in feet;

m = number of panels that are fully loaded to produce the maximum shear;

n = number of panels in truss.

EXAMPLE.—A six-panel truss like that represented in Fig. 26 has a panel length of 15 feet and carries a uniform live load of 1,200 pounds per linear foot of bridge. Compute the maximum positive live-load shear in the second panel from the left, or panel *bc*, Fig. 26.

SOLUTION.—The uniform load w'' per lin. ft. for one truss is $\frac{1}{2}\times 1{,}200 = 600$ lb. Also, in formula 2, $p=15$ ft., $m=4$, and $n=6$. Hence, the required shear is

$$V''=w''p\times\frac{m^2}{2(n-1)}=600\times 15\times\frac{4^2}{2\times(6-1)}=14{,}400 \text{ lb.} \quad \text{Ans.}$$

62. Approximate Live-Load Shears.—When a truss is loaded as shown in Fig. 26, the total load that is transmitted to the joint *c* is a little less than a full panel load, and some of the load in the panel *bc* is transmitted to joint *b*. However, the assumed load per linear foot is based on a rough estimate of the probable loading to which the bridge will be subjected, and extreme refinement in the computation of stresses is not warranted. Hence, it is customary to compute the live-load shear in any panel of a truss on the assumptions that every panel point on one side of the panel under consideration transmits a full panel live load and the panel points on the other side do not transmit any live load. The values of the shears that are based on these assumptions are somewhat greater than the values calculated by the formula of the preceding article, the error being greatest in the panels near the center of the span and zero in the end panels. Since the effect of the approximation is to obtain higher stresses in the web members near the center of the span, which are comparatively small, the approximate method has the advantage of providing additional safety as well as being more convenient.

EXAMPLE.—Solve the example of the preceding article by the approximate method.

SOLUTION.—A full panel load for one truss is

$$\frac{1{,}200\times 15}{2}=9{,}000 \text{ lb.}$$

According to the approximate method, it is assumed that there is a panel load of 9,000 lb. at each of the joints *c*, *d*, *e*, and *f* in Fig. 26 and no load at joint *b*. Hence, the shear in the panel *bc* is equal to the left-hand reaction of the truss for those four panel loads, or

$$V''=R_1''=\frac{9{,}000\times 1+9{,}000\times 2+9{,}000\times 3+9{,}000\times 4}{6}$$

$$=\frac{9{,}000\times(1+2+3+4)}{6}=15{,}000\text{ lb.}\quad\text{Ans.}$$

63. Maximum and Minimum Shears.—The maximum shear, or *maximum combined shear*, in a given panel of a truss is the sum of the dead-load shear and the maximum live-load shear of the same character as the dead-load shear. The minimum shear, or *minimum combined shear*, is equal to the difference between the dead-load shear and the maximum live-load shear of opposite character.

In general, the maximum live-load shear of the same character as the dead-load shear is produced in a given panel of a truss when every panel point between the panel in question and the more distant end of the truss carries a full live load and the panel points on the other side of the panel receive no live load. Thus, in Fig. 25, the dead-load shear in panel *bc* is positive, and the maximum positive live-load shear in that panel is obtained when there is a full panel live load at each of the joints *c*, *d*, *e*, and *f* to the right of the panel and no live load at the joint *b* to the left. The maximum live-load shear of opposite character to that of the dead-load shear is obtained in a panel when there is a full panel live load at each joint between the panel and the nearer end of the truss and no live load on the other joints of the truss. Thus, the maximum negative live-load shear in panel *bc* occurs when the live load is applied only at the joint *b*.

EXAMPLES FOR PRACTICE

1. A seven-panel bridge, each panel of which is 18 feet long, carries a dead load of 900 pounds per linear foot. Compute for one truss (*a*) the dead-load shear in the third panel from the left-hand end of the bridge and (*b*) the dead-load moment at the third panel point from the left-hand end.

Ans. { (*a*) 8,100 lb.
(*b*) 874,800 ft.-lb.

2. If the live load on the bridge in the preceding example is 1,500 pounds per linear foot, what is the maximum positive live-load shear for one truss in the third panel from the left end? Use the approximate method. Ans. 19,290 lb.

3. Solve the preceding example by means of formula 2, Art. 61. Ans. 18,000 lb.

STRESSES IN TRUSS MEMBERS

GENERAL PRINCIPLES

64. Outline of Method for Determining Stresses. When the panel loads on a truss and the reactions have been established, the stresses in the members of the truss can usually be determined by applying the principles of statics. In general, the procedure in finding the stress in a given member may be outlined as follows: First, the truss is imagined to be cut into two parts along a suitable section that passes through the member in which the stress is desired. Then, either part of the truss is treated as a free body which is in equilibrium under the action of a system of forces comprising (1) the reactions and loads applied to that part and (2) external forces representing the stresses in the members that are cut by the section. Finally, the required stress can be found by applying the conditions of equilibrium to that system of forces.

65. Method of Joints and Method of Sections.—There are two general methods of passing a section through a truss so as to divide the truss into two parts. In one method, all the members cut by the section are concurrent, that is, they intersect at a common joint of the truss. For example, in order to find the stress in a certain member of the truss shown in Fig. 27, it may be convenient to pass the curved section *GFI*, which cuts all the members that meet at the joint *B*, including the member whose stress is desired. The part of the

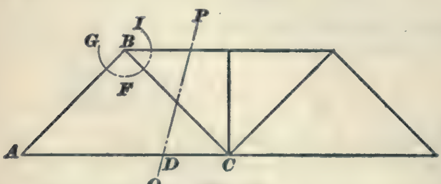


FIG. 27

truss between the joint B and the section GFI is then treated as a free body. In the second method, a section, as PQ in Fig. 27, is passed so as to cut both chords. In this case, the part of the truss on either side of the cutting section is treated as a free body.

When a stress in a truss member is determined by considering the forces acting on the part of the truss around a single joint, the stress is said to be found by the method of joints. If it is determined by considering the forces acting on the part of the truss to one side of a section that cuts both chords, the stress is said to be determined by the method of sections.

66. Analytic and Graphic Methods.—Stresses in truss members may be determined either by calculation or by means of diagrams; that is, either analytically or graphically. Since extreme accuracy in the results is not usually required, the choice of method is mainly a matter of convenience. The graphic method is generally preferred for finding the stresses in a structure, such as a roof truss, where the same conditions of loading are assumed for determining the maximum stresses in all the members. The analytic method, however, is more advantageous in the case of a bridge truss, because several different conditions of loading must be assumed in order to find the maximum and minimum stresses in the various web members.

67. Conditions of Equilibrium.—The forces of a balanced system, or a system that holds a body or a part of a body in equilibrium, must be related so as to fulfil the following three conditions: (1) The algebraic sum of the horizontal components of the forces is equal to zero; (2) the algebraic sum of their vertical components is equal to zero; (3) the algebraic sum of their moments about any point is equal to zero. When the stresses in a truss are to be calculated, the conditions of equilibrium are usually expressed by the following equations:

$$\Sigma X = 0 \qquad (1)$$

$$\Sigma Y = 0 \qquad (2)$$

$$\Sigma M = 0 \qquad (3)$$

in which ΣX denotes the algebraic sum of the horizontal components, ΣY is the algebraic sum of the vertical components, and ΣM is the algebraic sum of the moments.

If the stresses are to be found graphically, only the first two conditions of equilibrium are utilized. Equilibrium is then indicated by the fact that the vectors of the forces, or the lines representing the forces in both amount and direction, form a closed figure; in other words, the polygon of forces closes.

68. Relation Among a Force and Its Components. A force and its horizontal and vertical components may be considered to form a right triangle, as triangle *ABC* in Fig. 28, where *S* represents any force and *X* and *Y* represent its horizontal and vertical components, respectively. Hence, when the amount of a force and the angle *H* that it makes with the horizontal are known, the horizontal and vertical components of the force may be readily found by the relations

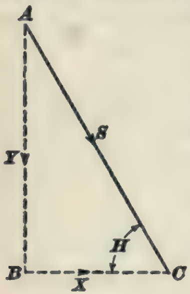


FIG. 28

$$X = S \cos H \qquad (1)$$

and

$$Y = S \sin H \qquad (2)$$

Also, if the direction of a force and the amount of one of its components are known, the amount of the force can be computed by one of the following formulas:

$$S = \frac{X}{\cos H} = X \sec H \qquad (3)$$

or

$$S = \frac{Y}{\sin H} = Y \csc H \qquad (4)$$

If the two components of the force are given, the angle that the force makes with the horizontal can be determined from the relation

$$\tan H = \frac{Y}{X} \qquad (5)$$

The amount of the force can then be computed by formula 3 or 4 or by the formula

$$S = \sqrt{X^2 + Y^2} \qquad (6)$$

69. Representation of Stresses by External Forces. When a part of a truss is treated as a free body that is in equi-

librium under the action of a system of forces, the stresses in the cut members are represented by external forces which have the same combined effect as the other part of the truss. These forces obviously must act along the axes of the cut members.

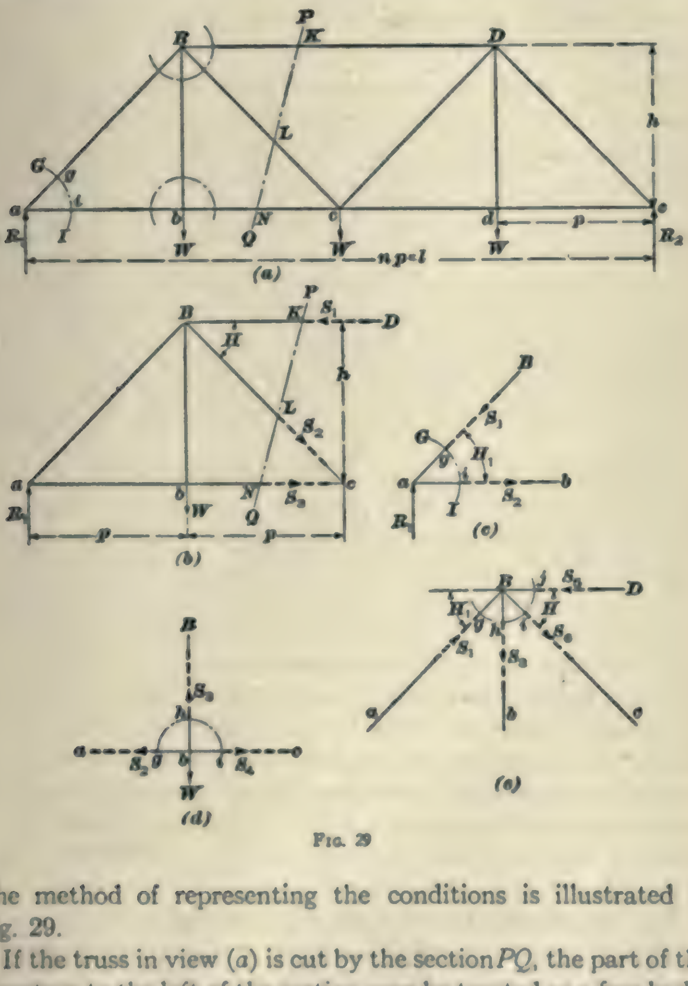


FIG. 29

The method of representing the conditions is illustrated in Fig. 29.

If the truss in view (*a*) is cut by the section *PQ*, the part of the structure to the left of the section may be treated as a free body, as shown in view (*b*). The stresses in the cut members *BD*, *Bc*, and *bc* are then represented by the forces S_1, S_2, and S_3, respec-

tively, which are directed along the members and are located on the same side of the section as the part of the truss they replace. Similarly, in views (*c*), (*d*), and (*e*) are shown the systems of forces that are acting on the parts of the truss around the joints *a*, *b*, and *B*, respectively. In these three views, and also in view (*a*), the cutting sections are indicated by dot-and-dash arcs.

When the stress in a member is represented by a force, the direction of the force along its line of action indicates whether the stress is tension or compression. It is usually assumed that a tensile stress, or pull, is indicated by a force acting away from the cutting section, and that a compressive stress, or thrust, is indicated by a force acting toward the cutting section. Thus, in Fig. 29 (*b*), the force S_1 indicates compression in the member *BD*, the force S_2 indicates tension in the member *Bc*, and the force S_3 indicates tension in the member *bc*. Also, the forces S_1 and S_5 in view (*e*) indicate compression in members *aB* and *BD*, respectively, and the forces S_3 and S_6 indicate tension in *Bb* and *Bc*.

70. Signs of Stresses.—Instead of indicating the characters of the stresses in truss members by means of the words "tension" and "compression" or by means of abbreviations for those words, it is often convenient to use the signs + and −. Either of these signs may be assumed to indicate tension and the other to indicate compression, provided the same assumptions are used throughout the computations. The choice in any particular case depends mainly on the practice of the organization where the work is being done. In former years, it has been the more general practice in the United States to use + for compression and − for tension. However, it is now becoming more common to use + for tension and − for compression.

71. Statically Indeterminate Trusses.—In some types of trusses, such as the multiple-system trusses shown in Figs. 8, 9, and 10, it is not possible to determine the stresses in the members simply by applying the principles of statics. Instead, an assumption is then made in regard to the distribution of

stress among two or more members, or consideration is given to the principles of elasticity in addition to the principles of statics. A truss in which the stresses cannot be found by the principles of statics alone is said to be statically indeterminate.

METHOD OF JOINTS

ANALYTIC METHOD

72. Selection of Joints for Analysis.—The general procedure outlined in Art. 64 applies to both the method of joints and the method of sections. However, the details of the solution are somewhat different in the two cases. When the method of joints is used, the system of forces acting on the part of the truss that is treated as a free body is analyzed graphically by constructing a polygon of forces or analytically by applying either or both of the relations $\Sigma X = 0$ and $\Sigma Y = 0$. Therefore, in order that this method may be employed, the section must be passed so as to cut not more than two members with unknown stresses, and one of these must be the member whose stress is desired. The order in which the joints are considered depends on this requirement. Because of the use of the relations $\Sigma X = 0$ and $\Sigma Y = 0$, the method of joints is sometimes called the *method of resolution of forces*.

73. Calculation of Stresses.—When the stresses in the members of a truss are to be determined analytically by the method of joints, the first step is to draw a skeleton diagram of the truss and to show all the loads and the reactions in their proper positions and directions. This diagram need not be to scale. The next step is to select a joint at which there are only two members whose stresses are unknown, and to pass a section around that joint. If desired, the part of the truss that is to be treated as a free body may be redrawn, but the entire solution is generally carried out by using the skeleton diagram alone. In order to make the following explanations clearer, the various parts of the truss will here be redrawn, as shown in Fig. 29 (*c*), (*d*), and (*e*).

After the cut part of the truss is redrawn, the stresses in the cut members are represented by external forces acting on the free body. Unless it is known from previous calculations or experience whether the stress in a certain member is tension or compression, the direction of each such force along its line of action is first assumed and later verified. In the usual procedure, the directions of the unknown forces are assumed arbitrarily in determining the directions of their components. Then, the values of the forces are computed by applying the conditions of equilibrium and solving the resulting equations. If the value obtained for a force is positive, the direction assumed for the force is correct. But, if the value is negative, the correct direction is opposite to that assumed.

EXAMPLE.—In a four-panel truss of the type illustrated in Fig. 29 (a), the panel length p is 16 feet, the height h is 14 feet, and the panel dead load W is 8,000 pounds. Calculate the dead-load stresses in the various members of the truss by the method of joints.

SOLUTION.—Since the truss in Fig. 29 is symmetrical, each reaction is equal to $\frac{8{,}000\times 3}{2}=12{,}000$ lb. Also, the stresses in the corresponding members on opposite sides of the center line are equal, and it is therefore sufficient to calculate the values for only one half of the truss.

Analysis of Joint a: Since the stresses in all members are unknown, it is necessary to analyze first the joint a, because that joint is the only one at which there are not more than two members. The section GI is passed around this joint and the part of the truss that is to be treated as a free body is redrawn in view (c). Here, the stresses in the cut members aB and ab are represented by the external forces S_1 and S_2, respectively, which are arbitrarily assumed to have the directions indicated by the arrowheads. Then, the forces acting on the part of the truss under consideration are the reaction R_1, which is known, and the forces S_1 and S_2, which are to be determined.

In order to apply the relations $\Sigma X=0$ and $\Sigma Y=0$, it is necessary to compute the components of the forces. As the reaction is vertical, its horizontal component is 0 and its vertical component is 12,000 lb. The horizontal component of the force S_2 is equal to the force itself, and its vertical component is 0. Also, the horizontal component of S_1 is $S_1 \cos H_1$ and the vertical component is $S_1 \sin H_1$. The angle H_1 may be found from the given panel length and the height of truss. Thus, in Fig. 29 (a),

$$\tan Bab=\frac{Bb}{ab}=\frac{14}{16} \text{ and } Bab=H_1=41° \ 11'$$

Wherever possible, the computations should be simplified by applying first the condition of equilibrium that will involve only one unknown force. In this case, the vertical component of force S_2 is zero, and the equation $\Sigma Y = 0$ will be applied. If upward forces or components are considered to be positive and downward forces or components are considered negative,

$$12{,}000 - S_1 \sin 41° \, 11' = 0$$

and

$$S_1 = \frac{12{,}000}{\sin 41° \, 11'} = 18{,}220 \text{ lb.}$$

Since this result comes out positive, the assumed direction of S_1 is correct. As that direction is toward the cutting section, the force S_1 represents a compressive stress. Hence, the stress in member aB or eD is 18,220 lb., compression. Ans.

After the value of the force S_1 has been determined, the other force S_2 can be readily computed by means of the relation $\Sigma X = 0$. If forces or components acting toward the right are considered positive and those to the left are considered negative, the equation is

$$-18{,}220 \cos 41° \, 11' + S_2 = 0$$

and

$$S_2 = 18{,}220 \cos 41° \, 11' = 13{,}710 \text{ lb.}$$

This result is also positive and therefore the direction of S_2 is away from the section, as assumed, and the stress in member ab or ed is 13,710 lb., tension. Ans.

Analysis of Joint b: After the stress in member ab has been found, there are only two members at joint b in which the stresses are unknown. Therefore, that joint will be analyzed next. The part of the truss included within a section passed around that joint is redrawn in Fig. 29 (d). This part is acted upon by four forces: namely, the panel load W; the force S_2 representing the stress in member ab, which is now known; and the forces S_3 and S_4, which represent the stresses in members bB and bc and are to be determined. Since the stress in ab is tension, the force S_2 is directed away from the cutting section. The directions of S_3 and S_4 are at first arbitrarily assumed to be as indicated by the arrows.

Here, the only vertical forces or components are the panel load W and the force S_3; and the only horizontal forces are S_2 and S_4. In each case, there is one known force and one unknown force, and the equations $\Sigma X = 0$ and $\Sigma Y = 0$ may be applied in either sequence. From the relation $\Sigma Y = 0$,

$$-8{,}000 + S_3 = 0$$

and

$$S_3 = 8{,}000 \text{ lb.}$$

Since the result is positive, the force S_3 acts away from the section, as assumed, and the stress in member bB or dD is 8,000 lb., tension. Ans.

From the relation $\Sigma X = 0$,

$$-13{,}710 + S_4 = 0$$

and

$$S_4 = 13{,}710 \text{ lb.}$$

Thus, the force S_4 also acts away from the cutting section and the stress in member bc or cd is 13,710 lb., tension. Ans.

Analysis of Joint B: Now that the stresses in the members aB and bB have been determined, the stresses in the other two members that meet at joint B, or members BD and Bc, can be computed by analyzing this joint. The part of the truss that would be removed if a section were passed around joint B is redrawn in Fig. 29 (e), and the forces representing the stresses in the cut members are given the directions indicated by the arrows. The directions of the known forces S_1 and S_3 are established by the characters of the stresses in the members aB and bB, and the directions of S_5 and S_6 are assumed.

Since the unknown force S_5 has no vertical component, the value of S_6 can be determined directly from the relation $\Sigma Y=0$. In this case, the angle DBc, Fig. 29 (a), is equal to the angle Bab, and the angles H_1 and H in view (e) are each equal to 41° 11′. Then,

$$S_1 \sin H_1 - S_3 - S_6 \sin H = 0$$

or

$$18{,}220 \sin 41° 11' - 8{,}000 - S_6 \sin 41° 11' = 0$$

and

$$S_6 = 6{,}070 \text{ lb.}$$

The force S_6 therefore acts away from the cutting section, and the stress in member Bc or cD is 6,070 lb., tension. Ans.

Finally, from the relation $\Sigma X=0$,

$$S_1 \cos 41° 11' - S_5 + S_6 \cos 41° 11' = 0$$

and

$$S_5 = 18{,}280 \text{ lb.}$$

Thus, the force S_5 acts toward the section and the stress in member BD is 18,280 lb., compression. Ans.

GRAPHIC METHOD

74. Construction of Stress Diagram.—The stresses in the members of a truss may be determined graphically by selecting joints for analysis in the manner described in Art. 72 and constructing the polygon of forces for each joint. However, in the case of a truss, it is customary to combine the various force polygons into a single diagram known as a stress diagram. In this way, duplication of lines is entirely avoided and considerable time is saved. The method of determining stresses by means of a stress diagram may be outlined as follows:

The first step is to draw to a convenient scale a frame diagram, such as that illustrated in Fig. 30 (a), which is an accurate outline of the truss showing all the members, reactions, and loads in their proper relative positions and directions. On this diagram, the external forces and the members are designated by letters or numbers placed in the spaces on each side of the lines rather than at the ends of the lines. Thus, the load W_2 in Fig. 30 (a) is called load 2-3 because it lies between the spaces marked 2

and 3, and the member between the spaces 11 and 12 is called 11-12. The space 0 includes all the area above the truss, as there are no loads at the top-chord joints.

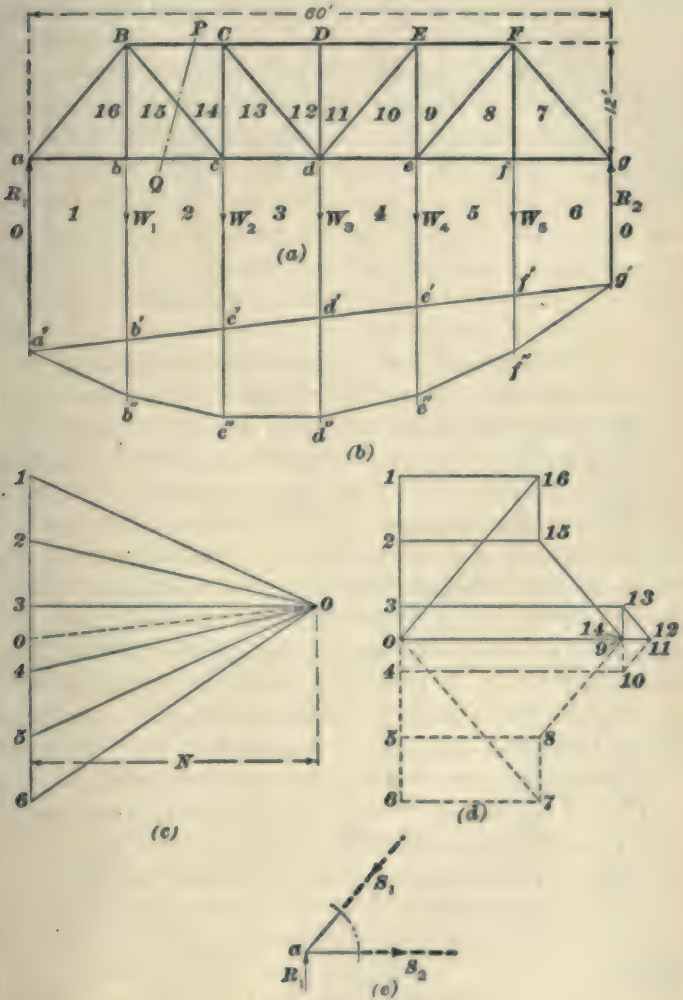


Fig. 30

After the frame diagram has been laid out, the stress diagram is begun by drawing to scale the polygon of forces for the loads

and reactions. In Fig. 30 (*d*), the vectors of the five given loads are laid off in succession along the vertical line from 1 to 6, and the points of division between the vectors are numbered to correspond to the spaces in view (*a*) between which the respective loads lie. Also, the point 0 in view (*d*) is located so that the lines 6-0 and 0-1 represent the reactions. In this case, the truss is symmetrically loaded and point 0 is midway between 1 and 6. The stress diagram is completed by analyzing the various joints of the truss in such order that there will be only two members with unknown stresses at the joint under consideration at the time. The polygon of forces for each joint may then be readily closed by drawing two lines that are parallel, respectively, to the members whose stresses are unknown.

75. The construction of the stress diagram in Fig. 30 (*d*) would be continued from the line 1-6 in the following manner: First, the triangle of forces for the joint *a* is completed by drawing a line from point 0 parallel to member *aB* or 0-16 in view (*a*) and another line from point 1 parallel to member *ab* or 1-16; the intersection of these lines is marked 16. The next step is to complete the polygon of forces for joint *b* by drawing a line from point 16 parallel to member 16-15 and a line from point 2 parallel to member 2-15, and marking 15 at their intersection. Similarly, point 14 is located by analyzing joint *B* and drawing lines from points 0 and 15 parallel to members 0-14 and 15-14, respectively. Then, point 13 is established at the intersection of lines 14-13 and 3-13, which are parallel to the members of the truss designated by the corresponding numbers; and point 12 is located by drawing lines 13-12 and 0-12. Theoretically, point 11 would be located at the intersection of a line drawn from point 12 parallel to member 12-11 and a line drawn from point 0 parallel to member 0-11. However, since the points 0 and 12 lie on the same horizontal line, point 11 must coincide with point 12.

In the case of a symmetrically-loaded truss, such as that shown in view (*a*), it is sufficient to construct the stress diagram for only one half of the truss. For this reason, the part of the stress diagram that would apply to the right-hand half is shown

dotted in view (*d*); this part is given merely to illustrate the fact that it is like the part for the left half of the truss.

76. Stresses From Stress Diagram.—The amount of the stress in any member of a truss is represented by the length of the line in the stress diagram between the points having the same numbers or letters as the spaces between which the member lies in the frame diagram. For example, the amount of the stress in the member *Bc*, or 15-14, in Fig. 30 (*a*) is indicated by the length of the line 15-14 in view (*d*) measured to the same scale that was used for laying off the vectors of the loads along the line 1-6.

The character of the stress in a member may also be determined from the stress diagram as follows: First, consider a section cutting the member and passing around a joint that supports an external load; for instance, to find the character of the stress in member *Bc*, a section would be passed around joint *c*, at which the load W_2 is supported. From the direction in which that load acts, determine in what order the designating figures or letters for that load should be read in the stress diagram and also in what direction—clockwise or counter-clockwise—it is necessary to proceed around the joints in the frame diagram. Thus, since the load W_2 acts downwards, it is designated in the stress diagram as 2-3 (not 3-2), and obviously the direction of procedure around the joint *c* in the frame diagram is counter-clockwise. This manner of proceeding around the joint will indicate how to read the line in the stress diagram that represents the desired stress and will determine the direction in which the force representing that stress acts with respect to the joint. By proceeding in the counter-clockwise direction around joint *c* in view (*a*), it is found that the designation for member *Bc* should be 14-15. If the line corresponding to the stress in *Bc* is located in the stress diagram in view (*d*), and read 14-15, it is obvious that the force representing the stress acts diagonally upwards, or away from the joint *c* in view (*a*), and hence the stress in member *Bc* is tension.

As a further illustration of the manner in which the characters of stresses are determined, the stresses in members *aB* and *ab* will

be considered. Both members pass through joint *a*, and a section around that joint will be assumed. The reaction R_1 acts upwards and is designated in the stress diagram in view (*d*) as 0-1 (not 1-0), indicating that the direction of procedure around the joint *a* is to be counter-clockwise. Therefore, it is necessary to read the member *ab* as 1-16 and the member *aB* as 16-0. If the lines in the stress diagram corresponding to the stresses in these members are read in such order, it is seen that the force representing the stress in member *ab* acts to the right and the force for member *aB* acts diagonally downwards, as shown by the directions of the forces S_2 and S_1 in view (*e*). Thus, the force S_2 acts away from the joint *a* and the stress in member *ab* is tension; and the force S_1 acts toward the joint and the stress in member *aB* is compression.

METHOD OF SECTIONS

77. Analytic Method of Finding Stresses.—When the method of sections is to be used for finding the stresses in the members of a truss and the values are to be determined analytically, the section can usually be passed through any member whose stress is desired, and no particular order need be followed. In the case of a chord member, the section cutting the truss into two parts is generally passed through that member, a member of the opposite chord, and only one web member; and the relation $\Sigma M = 0$ is applied to the system of forces acting on the part of the truss that is to be treated as a free body. In order that the desired stress will be the only unknown quantity in the equation of moments, the center of moments is taken at the intersection of the other two members cut by the section. The moment of the stress in each of these other two members is then zero, and the value of the required stress can be readily found by solving the equation $\Sigma M = 0$. For example, if it is desired to determine the stress in the member *AC* in Fig. 27, a section *PQ* would be passed through that member, the top chord, and the web member *BC*; and the center of moments would be taken at the intersection of *BC* and the top chord, or at joint *B*.

If the character of the required stress cannot be determined by inspection, the direction of the force representing the stress is assumed arbitrarily in determining the sign of the moment of the force in the equation $\Sigma M=0$. This assumed direction is correct if the resulting value of the force is positive, and is incorrect if the result is negative.

78. The method of the preceding article can be applied also for finding the stress in a web member of an inclined-chord or curved-chord truss. In this case, however, the section would be passed through the web member whose stress is desired and through the top and bottom chords, and the center of moments would be taken at the intersection of the chords.

The stress in a web member of a parallel-chord truss cannot be computed by applying the relation $\Sigma M=0$ in the manner just described, because the top and bottom chords do not intersect. However, such a stress can be computed by the following simple method. A section is passed through the web member and the top and bottom chords, and the relation $\Sigma Y=0$ is applied to the system of forces acting on the part of the truss that is considered as a free body. Since both chord members are horizontal, neither of the forces representing the stresses in those members has a vertical component, and the only unknown quantity in the equation is the required stress in the web member. For instance, the stress in the member BC, Fig. 27, may be found by passing the section PQ and equating to zero the algebraic sum of the reaction at A; the panel load at B, if there is one; and the vertical component of the force representing the stress in BC. The character of the stress can be determined either by inspection or by first assuming the direction of the force representing the stress and then verifying the assumption, as described in the preceding article for a chord member.

The method of sections is sometimes called the *method of moments and shears*.

EXAMPLE.—For the truss considered in the example of Art. 73 and represented in Fig. 29 (*a*), compute the stresses in the members BD, bc, and Bc by the method of sections.

SOLUTION.—*Member BD:* To find the stress in the member BD, the first step is to pass the section PQ, view (*a*), through that member, the

lower chord, and the web member Bc. The part of the truss to the left of the section is then treated as a free body, and the equation $\Sigma M=0$ is applied. If desired, this part may be redrawn, as in view (*b*); and the stresses in the cut members may be represented by the forces S_1, S_2, and S_3, whose directions are at first assumed to be as indicated by the arrows. In order that the force S_1 will be the only unknown in the equation of moments, the center of moments is taken at the intersection of the members bc and Bc, or at joint c.

Since the reaction R_1 is 12,000 lb., the panel load W is 8,000 lb., the panel length p is 16 ft., and the height h is 14 ft., the equation of moments is

$$12{,}000\times 32-8{,}000\times 16-S_1\times 14=0$$

from which $$S_1=\frac{12{,}000\times 32-8{,}000\times 16}{14}=18{,}280 \text{ lb.}$$

The positive result indicates that the assumed direction of the force S_1 is correct. Hence, the stress in member BD is 18,280 lb., compression. Ans.

Member bc: The section PQ, Fig. 29 (*a*), is also satisfactory for determining the stress in the member bc. But, in this case, the center of moments is taken at the intersection of members BD and Bc, or at joint B. Then, the equation of moments is

$$12{,}000\times 16-S_3\times 14=0$$

and $$S_3=\frac{12{,}000\times 16}{14}=13{,}710 \text{ lb.}$$

This force acts away from the section, as assumed, and the stress in member bc is 13,710 lb., tension. Ans.

Member Bc: Since the section PQ cuts the member Bc and both chords, that section is suitable for determining the stress in the web member. In this case, the relation $\Sigma Y=0$ is applied to the system of forces shown in Fig. 29 (*b*). Thus,

$$12{,}000-8{,}000-S_2 \sin H=0$$

and, since $H=41° \ 11'$,

$$S_2=\frac{12{,}000-8{,}000}{\sin 41° \ 11'}=6{,}070 \text{ lb.}$$

The force S_2 acts away from the section and the stress in the member Bc is, therefore, 6,070 lb., tension. Ans.

79. Rules for Stresses in Parallel-Chord Trusses. The stresses in the members of a parallel-chord truss can be computed most easily by applying the following rules, which are based on the explanations in the preceding two articles.

To find the stress in any chord member of a parallel-chord truss, pass a section through the member and the opposite chord, and locate the center of moments, as described in Art. 77;

then divide the moment about the center of moments by the height of the truss.

To find the stress in any web member of a parallel-chord truss, pass a section through the member, as described in Art. 78; then divide the shear on the section by the sine of the angle that the web member makes with the horizontal.

80. Graphic Determination of Shears and Moments. The shear on any section through a truss or the moment at any panel point may be determined graphically by constructing an equilibrium polygon, as in Fig. 30 (*b*), and a load line and rays from a convenient pole, as in view (*c*). The shear on any section is equal to the distance on the load line in view (*c*) from the point 0 to the lower extremity of the vector of the load that lies just to the left of the section. Thus, the shear on section *PQ*, view (*a*), is equal to the distance in view (*c*) from point 0 to point 2, which is the lower extremity of the vector of load W_1 in view (*a*).

To find the moment at any panel point, it is simply necessary to take the product of the horizontal distance *N* from the pole to the load line and the vertical distance intercepted by the equilibrium polygon on a vertical line through the given panel point. Thus, the moment at joint *d*, Fig. 30 (*a*), is equal to the product of the distances *N* in view (*c*) and *d′d″* in view (*b*).

STRESSES IN BRIDGE TRUSSES

(PART 2)

Serial 943B | Edition 1

PARALLEL-CHORD TRUSSES

THE SINGLE-SYSTEM WARREN TRUSS

INTRODUCTION

1. Description.—The **Warren truss**, Fig. 1, is a simple type of truss with parallel chords, in which the web members are all inclined and make the same angle with the vertical, giving the truss the appearance of a series of connected isosceles triangles; it is sometimes called the *triangular*

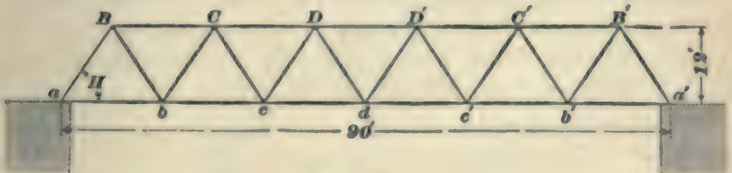


Fig. 1

truss. The Warren truss is used in deck, through, and half-through bridges, is more frequently built as a riveted than as a pin-connected truss, and is especially adapted to the shorter spans for which trusses are used. For spans up to about 100 feet, it is frequently spoken of as a *lattice girder*. For longer spans, it is sometimes built with subdivided panels, or with multiple systems of web members.

2. Methods of Calculation.—The stresses in the members of the simple type of Warren truss can be readily found, either graphically or analytically, by applying the general conditions of equilibrium. The work of calculation by the analytic methods is so simple that the graphic method is seldom used in practice for this type of truss.

The analytic methods are illustrated in the following articles, which contain the calculations of the maximum and minimum stresses in all the members of the six-panel truss shown in Fig. 1. This truss has a span of 90 feet and a height of 12 feet; the dead load is taken as 600 pounds, and the live load as 1,600 pounds, per linear foot of the bridge; all the dead load is assumed to be applied at the joints of the loaded chord, and the truss is assumed to support one-half the entire load on the bridge.

METHOD OF SECTIONS

3. Panel Loads and Reactions.—The dead panel load W' for one truss is equal to $\frac{600}{2} \times 15 = 4{,}500$ pounds. As explained in *Stresses in Bridge Trusses*, Part 1, the number of panel loads considered in determining the reactions is one less than the number of panels in the truss. In this case, the number of panels in the truss is six; therefore, only five panel loads are taken into account in determining the reactions. The reactions R_1' and R_2', Fig. 2 (a), due to the dead load are each equal to

$$\frac{4{,}500 \times 5}{2} = 11{,}250 \text{ pounds}$$

The live panel load W'' for one truss is equal to $\frac{1{,}600}{2} \times 15 = 12{,}000$ pounds; and the reactions R_1'' and R_2'' for a fully loaded truss are each equal to

$$\frac{12{,}000 \times 5}{2} = 30{,}000 \text{ pounds}$$

4. Chord Stresses in General.—Chord stresses may be conveniently determined by the method of sections

explained in *Stresses in Bridge Trusses*, Part 1. The loads and reactions for a typical truss with dead loads are shown in Fig. 2 (*a*). The method will be illustrated by determining the dead-load stresses in *CD* and *cd*. The truss may be considered cut by a plane *q* intersecting the members *CD*, *cD*, and *cd*. The portion of the truss to the left of section *q* is shown in Fig. 2 (*b*), the external forces being R_1' at *a*,

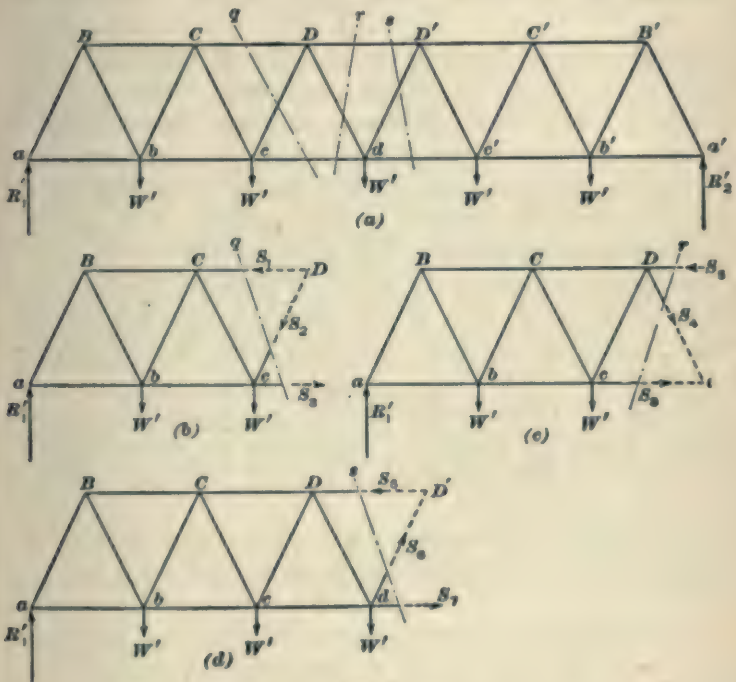


FIG. 2

W' at *b*, W' at *c*, and the forces S_1, S_2, and S_3, equal numerically to the stresses in the members cut.

Assuming that the stress in *CD* is compression, S_1 will be directed toward the left, and its magnitude may be computed by taking moments of all the forces about *c*, the point of intersection of S_2 and S_3. The moments of the load at *c* and of S_2 and S_3 are each equal to zero, since their lever arms are zero. Writing the equation for the

moments of all the forces shown in Fig. 2 (*b*) about the point *c*, we have

$$R_1' \times 30 - W' \times 15 - S_1 \times 12 = 0;$$

whence

$$S_1 = \frac{R_1' \times 30 - W' \times 15}{12} = \frac{\frac{5W'}{2} \times 30 - W' \times 15}{12}$$

$$= \frac{4W' \times 15}{12}$$

As the value of S_1 comes out positive, the assumption that the stress in CD is compression is correct. The advantage of taking *c* as a center of moments is that, by so doing, an equation is obtained that contains but one unknown force, namely, S_1; the other two, S_2 and S_3, do not appear in the equation, since the moment of each is zero. The center of moments may be taken at any point, whether it is on the structure or not, but it is better, if possible, to take it at the intersection of two of the members, thereby eliminating the stresses in those members from the equation of moments.

To find S_3, the center of moments may be taken at the intersection D of S_1 and S_2. Assuming the stress in $c\,d$ to be tension, S_3 will be directed toward the right. Taking moments about D,

$$R_1' \times 37.5 - W' \times 22.5 - W' \times 7.5 - S_3 \times 12 = 0;$$

whence $$S_3 = \frac{R_1' \times 37.5 - W' \times 22.5 - W' \times 7.5}{12}$$

If this comes out positive, the assumption that the stress in $c\,d$ is tension is correct; if negative, the stress is compression, but its numerical value will be that determined by the last equation.

In a similar manner, if the stress in $D\,D'$ is required, the truss may be considered cut by a plane *r*, intersecting $D\,D'$, $D\,d$, and $c\,d$, or by a plane *s*, intersecting $D\,D'$, $d\,D'$, and $d\,c'$. The portion to the left of section *r* is shown in Fig. 2 (*c*); the portion to the left of section *s* is shown in Fig. 2 (*d*). The proper center of moments is *d*. The stress in $D\,D'$ will be assumed as compression; then, S_1 will be

directed toward the left. Writing the expression for the moment at d,

$$R_1' \times 45 - W' \times 30 - W' \times 15 - S_6 \times 12 = 0;$$

whence $$S_6 = \frac{R_1' \times 45 - W' \times 30 - W' \times 15}{12}$$

In Fig. 2 (c), the lever arms of S_3 and S_4 are each zero; in Fig. 2 (d), the lever arms of S_5 and S_7 and of the load at d are each zero; hence, these do not appear in the equation of moments.

The values of the other chord stresses can be found in a similar manner. All upper-chord members will be in compression and all lower-chord members in tension.

5. It will be seen that the numerator of the expression for the stress in any chord member is, in each case, the sum of the moments of the panel loads and reactions at the left of the section, about the joint opposite the member. This sum is the bending moment on the truss at that point. The denominator is the height of the truss. We may, therefore, state the following general principle:

The stress in any chord member of a simple Warren truss is equal to the bending moment on the truss, at the joint opposite the member considered, divided by the height of the truss.

6. Dead-Load Chord Stresses.—Applying the principle just stated to the determination of the dead-load chord stresses, the following values are found:

$$\text{Stress in } a\,b = \frac{\text{moment at } B}{\text{height}} = \frac{11{,}250 \times 7.5}{12}$$
$$= 7{,}030 \text{ pounds, tension.}$$

$$\text{Stress in } b\,c = \frac{\text{moment at } C}{\text{height}} = \frac{11{,}250 \times 22.5 - 4{,}500 \times 7.5}{12}$$
$$= 18{,}280 \text{ pounds, tension.}$$

$$\text{Stress in } c\,d = \frac{\text{moment at } D}{\text{height}}$$
$$= \frac{11{,}250 \times 37.5 - 4{,}500 \times 22.5 - 4{,}500 \times 7.5}{12}$$
$$= 23{,}910 \text{ pounds, tension.}$$

$$\text{Stress in } BC = \frac{\text{moment at } b}{\text{height}} = \frac{11{,}250 \times 15}{12}$$
$$= 14{,}060 \text{ pounds, compression.}$$

$$\text{Stress in } CD = \frac{\text{moment at } c}{\text{height}} = \frac{11{,}250 \times 30 - 4{,}500 \times 15}{12}$$
$$= 22{,}500 \text{ pounds, compression.}$$

$$\text{Stress in } DD' = \frac{\text{moment at } d}{\text{height}}$$
$$= \frac{11{,}250 \times 45 - 4{,}500 \times 30 - 4{,}500 \times 15}{12}$$
$$= 25{,}310 \text{ pounds, compression.}$$

As the truss is symmetrical, the stresses in the members on the right of the center are equal to those in the corresponding members on the left. That is, the stress in $D'C'$ is equal to the stress in CD; the stress in $C'B'$ is equal to the stress in BC; etc.

7. Live-Load Chord Stresses.—The maximum bending moments, and, therefore, the maximum chord stresses, due to a moving load, occur when the truss is fully loaded. This condition of loading is similar to the dead loading, each panel load being now 12,000 pounds, and each reaction 30,000 pounds, in place of 4,500 and 11,250 pounds, respectively. The chord stresses may be found in precisely the same way as for dead loads; thus,

$$\text{stress in } ab = \frac{30{,}000 \times 7.5}{12} = 18{,}750 \text{ pounds}$$

and so on. The results are (using the minus sign for tension and the plus sign for compression):

Member	Stress, in Pounds
ab	$-18{,}750$
bc	$-48{,}750$
cd	$-63{,}750$
BC	$+37{,}500$
CD	$+60{,}000$
DD'	$+67{,}500$

8. As all the dead load is assumed as being applied at the joints of the loaded chord, the live-load stresses in the

chords may be obtained from the dead-load stresses by multiplying the latter by the ratio of the live to the dead load per linear foot, which ratio is $\frac{1,600}{600}$, or $\frac{8}{3}$. For example, the dead-load stress in bc is $-18,280$, and the live-load stress is

$$-18,280 \times \frac{8}{3} = -48,750 \text{ pounds}$$

9. Maximum and Minimum Chord Stresses.—Since the live-load stress in any chord member is of the same sign as the dead-load stress, the maximum stress in the member is equal to the sum of the two stresses; and the minimum stress is equal to the dead-load stress.

10. Web Stresses in General.—The stresses in the web members may be found by the method of shears, explained in *Stresses in Bridge Trusses*, Part 1. For example, to determine the stress in the web member cD, Fig. 2 (a), the portion of the structure to the left of section q may be considered as a free body, as shown in Fig. 2 (b). Any of the conditions of equilibrium may be applied to the forces shown. It is desirable, if possible, to use an equation that contains the web force S_2, to be determined, but which does not involve either of the two forces S_1 and S_3. As S_1 and S_3 are horizontal, they will not appear in the equation $\Sigma Y = \Sigma S \sin H = 0$. Assuming the stress in cD to be compression, S_2 will act downwards to the left. Writing the expression for $\Sigma Y = \Sigma S \sin H = 0$ gives

$$\Sigma Y = R_1' - W' - W' - \text{vertical component of } S_2 = 0;$$

that is, denoting the angle DcS_2 by H,

$$R_1' - 2W' - S_2 \sin H = 0;$$

whence $$S_2 \sin H = R_1' - 2W'$$

and $$S_2 = \frac{R_1' - 2W'}{\sin H} = (R_1' - 2W') \csc H$$

The term $R_1' - 2W'$ is the shear on the section q; therefore, the vertical component $S_2 \sin H$ of S_2 is numerically equal to the shear on the plane of section that cuts cD. In general, the following principle may be stated:

For single-system parallel-chord trusses, the vertical component of the stress in any web member is numerically equal to the shear

on the plane of section cutting that web member and the two chord members between which the web member lies; and the stress in the same web member is numerically equal to the shear just referred to, multiplied by the cosecant of the angle that the member makes with the horizontal.

11. Character of Web Stresses.—If the shear on section q, Fig. 2 (b), is positive, the resultant of the external vertical forces on the left of the section acts upwards; then S_2 must act downwards, and the stress in cD is compression. If the shear is negative, S_2 acts upwards, and the stress in cD is tension. If the shear on section r, Fig. 2 (c), is positive, the resultant of the external forces on the left acts upwards, S_4 acts downwards, and the stress in Dd is tension. If the shear is negative, S_4 acts upwards, and the stress in Dd is compression. These conclusions may be stated as a general principle thus:

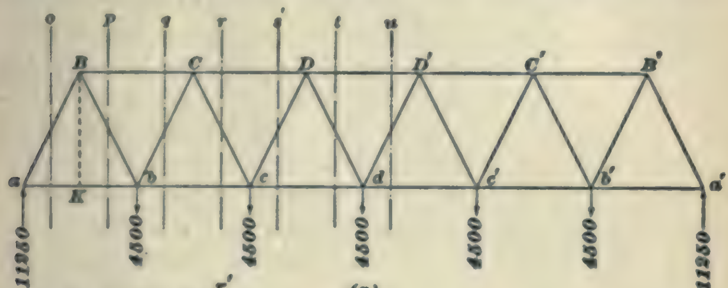
In those web members inclining downwards toward the left or upwards toward the right, positive shear causes compression, and negative shear tension; in those web members inclining upwards toward the left or downwards toward the right, positive shear causes tension, and negative shear compression.

12. Dead-Load Shears and Web Stresses.—In order to calculate the stresses in the web members due to dead load, it will be convenient first to find the shears on the sections cut by the planes o, p, q', etc., Fig. 3 (a). They are as follows:

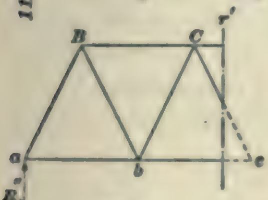
MEMBER	SECTION	SHEAR, IN POUNDS
aB	o	+ 11,250
Bb	p	+ 11,250
bC	q'	+ 6,750
Cc	r'	+ 6,750
cD	s'	+ 2,250
Dd	t	+ 2,250
dD'	u	− 2,250

From symmetry, the shears to the right of the center d will be equal and of opposite sign to the corresponding shears on the left. For example, the shear on section

t is $+$ 2,250 pounds, and that on section u is $-$ 2,250 pounds. The shears on the sections o and p are equal, as are also those on q' and r', and those on s' and t; because, in each case, the two planes are passed between the same two panel loads, that is, in the same panel of the loaded chord; and, as in the present case there are no loads applied at the joints of the unloaded chord, the shears on all sections in any panel are equal. As all shears to the left of d are positive, the stresses in the members $a\,B$, $b\,C$, and $c\,D$ that incline down-



(a)



(b)

FIG. 3

wards toward the left are compression, and the stresses in the members $B\,b$, $C\,c$, and $D\,d$ that incline upwards toward the left are tension. On the right of the center, the shears are negative; and the stresses in the members that incline downwards toward the left are tension, while the stresses in those that incline upwards toward the left are compression. In the present case, the stresses in $a\,B$ and $B\,b$ are equal and of opposite signs, as are also those in $b\,C$ and $C\,c$, etc.

Referring to Fig. 3 (a), and applying the general principle given in Art. **10**, we have

$$\csc H = \csc B\,a\,K = \frac{B\,a}{B\,K} = \frac{\sqrt{\overline{B\,K}^2 + \overline{a\,K}^2}}{B\,K}$$

$$= \frac{\sqrt{\overline{B\,K}^2 + \left(\frac{a\,b}{2}\right)^2}}{B\,K} = \frac{\sqrt{12^2 + 7.5^2}}{12} = 1.18$$

The web stresses can now be computed. They are as follows:

MEMBER	STRESS, IN POUNDS
aB, $a'B'$	$11{,}250 \times 1.18 = +13{,}280$
Bb, $B'b'$	$11{,}250 \times 1.18 = -13{,}280$
bC, $b'C'$	$6{,}750 \times 1.18 = +\ 7{,}970$
Cc, $C'c'$	$6{,}750 \times 1.18 = -\ 7{,}970$
cD, $c'D'$	$2{,}250 \times 1.18 = +\ 2{,}660$
Dd, $D'd$	$2{,}250 \times 1.18 = -\ 2{,}660$

13. Live-Load Shears and Web Stresses.—The stresses caused in the web members by the live load may be found from the shears. As the maximum stresses are desired, the truss must be so loaded as to cause the maximum shear for each case. The approximate method of loading explained in *Stresses in Bridge Trusses*, Part 1, will be used. The maximum positive shear in any panel occurs when all joints to the right of the panel are loaded; the maximum negative shear occurs when all joints to the left are loaded. Thus, in member Cc, Fig. 3 (a), the maximum tension occurs when all joints from c to b' are loaded; and the maximum compression occurs when the joint b is loaded. When joints c to b' are loaded, the left reaction is

$$\frac{12{,}000 \times (1 + 2 + 3 + 4)}{6} = 20{,}000 \text{ pounds}$$

As there is no load at b, the only force acting on the portion of the truss to the left of r' is the left reaction. Then, the shear in the panel bc is equal to the left reaction, Fig. 3 (b), or 20,000 poands. The stress in Cc is equal to the shear in panel bc multiplied by csc H; or,

$$\text{stress in } Cc = 20{,}000 \times 1.18 = 23{,}600 \text{ pounds, tension}$$

In like manner, the stress in any other member may be found. The maximum positive live shears are as follows:

PANEL	LOAD	SHEAR, IN POUNDS
ab	From b to b'	30,000
bc	From c to b'	20,000
cd	From d to b'	12,000
dc'	At c' and b'	6,000
$c'b'$	At b'	2,000

In the panel $b' a'$ there can be no positive shear.

The maximum negative shear in any panel is numerically equal to the maximum positive shear in the corresponding panel at the other end of the truss. The maximum and minimum stresses in the members can now be found by multiplying the respective shears by csc H. These stresses are given in the following table:

Panel	Member	Positive Shear Pounds	Stress Due to Positive Shear Pounds	Negative Shear Pounds	Stress Due to Negative Shear Pounds
$a b$	$a B$	30,000	+ 35,400		
$a b$	$B b$	30,000	− 35,400		
$b c$	$b C$	20,000	+ 23,600	2,000	− 2,360
$b c$	$C c$	20,000	− 23,600	2,000	+ 2,360
$c d$	$c D$	12,000	+ 14,160	6,000	− 7,080
$c d$	$D d$	12,000	− 14,160	6,000	+ 7,080

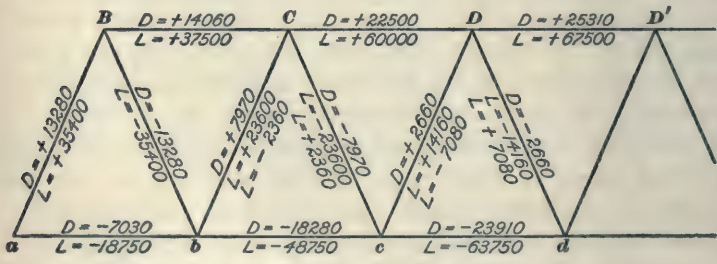
14. Combined Shears and Web Stresses.—The maximum and minimum stresses caused in the members on the left of the center by combined dead and live loads may be found by multiplying the maximum and minimum shears, respectively, by csc H. The maximum shear in any panel is equal to the sum of the positive dead-load and the positive live-load shear in the panel; the minimum shear is equal to the algebraic sum of the positive dead-load and the negative live-load shear in the panel. In columns 3 and 5 of the following table are given the maximum and minimum shears, respectively; while in columns 4 and 6 are given the maximum and minimum stresses, respectively, each stress being obtained by multiplying the corresponding shear by csc H.

In the members $a B$ and $B b$, the minimum stresses are equal to the dead-load stresses, as there can be no negative live-load shear in the panel $a b$. The minimum stresses in $c D$ and $D d$ are of opposite sign to the maximum, because

in the panel *c d* the negative live-load shear exceeds the positive dead-load shear. Under the special conditions here assumed, the combined shear is positive when the joints to the right of *d* are loaded, and negative when those to the

1	2	3	4	5	6
Panel	Member	Maximum Shear Pounds	Maximum Stress Pounds	Minimum Shear Pounds	Minimum Stress Pounds
a b	*a B*	+ 41,250	+ 48,680	+ 11,250	+ 13,280
a b	*B b*	+ 41,250	− 48,680	+ 11,250	− 13,280
b c	*b C*	+ 26,750	+ 31,570	+ 4,750	+ 5,610
b c	*C c*	+ 26,750	− 31,570	+ 4,750	− 5,610
c d	*c D*	+ 14,250	+ 16,820	− 3,750	− 4,420
c d	*D d*	+ 14,250	− 16,820	− 3,750	+ 4,420

left of *d* are loaded. This is an important point, and shows that, in the present case, the members *c D* and *D d* are sometimes in tension and sometimes in compression, according to the position of the live load. The two values of the stress given for each member are the extreme values that can



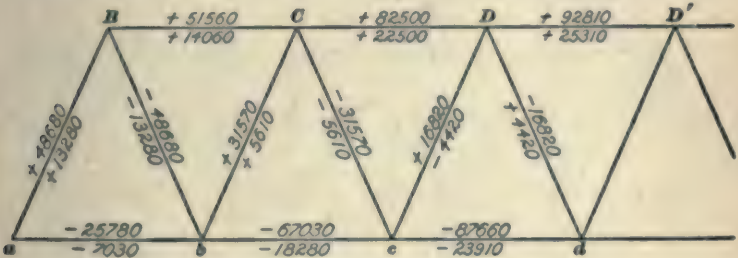
Dead and Live Load Stresses

FIG. 4

exist in that member for the given loads. In each of the members *a B*, *B b*, *b C*, and *C c*, the stress may have any value between the extreme values, and such stresses will always be of the same kind, that is, tension or compression. In each of the members *c D* and *D d*, the stress may have

any value between the positive and the negative value given. The stress in each member will reverse when the combined shear changes from positive to negative.

The stresses in all the members are shown in Figs. 4 and 5, which should be carefully studied. In Fig. 4, L represents



Maximum and Minimum Stresses

FIG. 5

the live-load stress, and D the dead-load stress. In Fig. 5, the maximum is placed above and the minimum below the line representing the member. Notice how the maximum and minimum stresses in Fig. 5 are obtained by addition from the stresses in Fig. 4.

METHOD OF JOINTS

15. For purposes of comparison, the maximum and minimum stresses in the example of Art. **2** will be calculated by the method of joints. The truss is represented in Fig. 6 (A), the dead panel load being 4,500 pounds, and the live panel load, 12,000 pounds. The figure gives

$$\cot H = \cot B\,a\,K = \frac{a\,K}{B\,K} = \frac{7.5}{12} = .625$$

and, as before (Art. **12**), $\csc H = 1.18$.

16. Dead-Load Stresses.—It will be convenient to start at joint a, as there are only two unknown stresses at that joint.

Joint a.—This joint is represented as a free body in Fig. 6 (a), the forces acting on it being the reaction $R_1' = 11{,}250$ pounds, and the forces S_1 and S_{11}, the last two

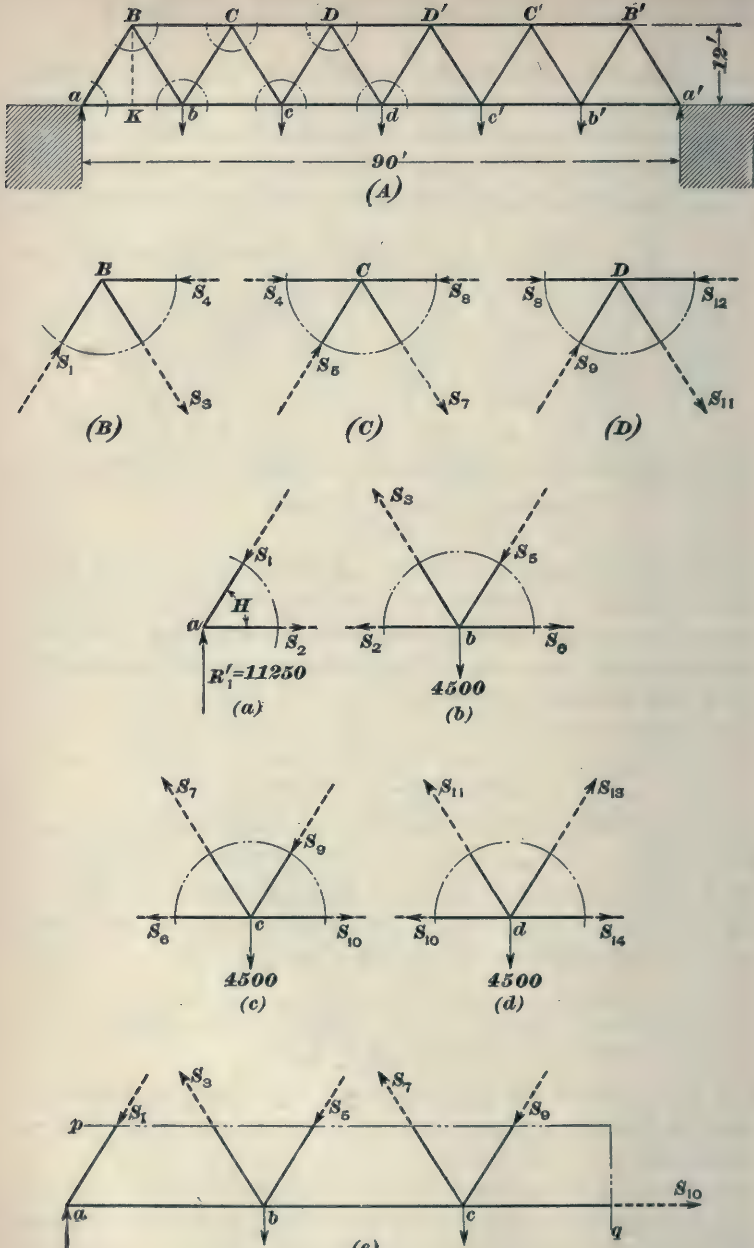

B
C
D
D′
C′
B′
12′
a
K
b
c
d
c′
b′
a′
90′
(A)
B
S4
S1
S3
(B)
C
S4
S8
S5
S7
(C)
D
S8
S12
S9
S11
(D)
S1
H
a
S2
R′1=11250
(a)
S3
S5
S2
b
S6
4500
(b)
S7
S9
S6
c
S10
4500
(c)
S11
S13
S10
d
S14
4500
(d)
S1
S3
S5
S7
S9
p
S10
a
b
c
q
(e)

Fig. 6

representing the stresses in aB and ab, respectively. The stress in aB will be assumed as compression, and that in ab as tension. Then, S_1 will act downwards to the left, and S_2 horizontally to the right. From the general conditions of equilibrium, we have, since the vertical component of S_2 is zero,

$$\Sigma Y = R_1' - S_1 \sin H = 0;$$

whence $$S_1 \sin H = R_1' = 11{,}250 \text{ pounds}$$

and

$$S_1 = 11{,}250 \csc H = 11{,}250 \times 1.18$$
$$= 13{,}280 \text{ pounds, compression in } aB$$

Likewise, since R_1' has no horizontal component,

$$\Sigma X = S_1 \cos H - S_2 = 0;$$

whence

$$S_2 = S_1 \cos H = 11{,}250 \csc H \cos H$$
$$= 11\ 250 \frac{\cos H}{\sin H}$$
$$= 11{,}250 \cot H = 11{,}250 \times .625$$
$$= 7{,}030 \text{ pounds, tension in } ab$$

Joint B.—This joint is represented in Fig. 6 (B), the forces acting on it being S_1, S_3, and S_4, which represent the stresses in aB, Bb, and BC, respectively. The force S_1 is known and acts upwards to the right; the stress in Bb will be assumed as tension, and that in BC as compression. Then, S_3 will act downwards to the right, and S_4 horizontally to the left. From the conditions of equilibrium,

$$\Sigma Y = S_1 \sin H - S_3 \sin H = 0;$$

whence $S_3 \sin H = S_1 \sin H = 11{,}250$ pounds

and $S_3 = 11{,}250 \csc H = 13{,}280$ pounds, tension in Bb

Likewise,

$$\Sigma X = S_1 \cos H + S_3 \cos H - S_4 = 0;$$

whence

$$S_4 = (S_1 + S_3) \cos H = (11{,}250 \csc H + 11{,}250 \csc H) \cos H$$
$$= 22{,}500 \csc H \cos H = 22{,}500 \cot H$$
$$= 14{,}060 \text{ pounds, compression in } BC$$

Joint b.—This joint is represented in Fig. 6 (b), the forces acting on it being S_2, S_3, S_5, and S_6, which represent the stresses in ab, Bb, bC, and bc, respectively; and the panel

load of 4,500 pounds. The latter acts vertically downwards, S_2 acts horizontally to the left, and S_3 acts upwards to the left; S_5 and S_6 are unknown. The stress in bC will be assumed as compression, and that in bc as tension. Then, S_5 will act downwards to the left and S_6 horizontally to the right.

$$\Sigma Y = S_3 \sin H - 4{,}500 - S_5 \sin H = 0;$$

whence

$$S_5 \sin H = S_3 \sin H - 4{,}500 = 11{,}250 - 4{,}500$$
$$= 6{,}750 \text{ pounds}$$

and

$$S_5 = 6{,}750 \csc H = 7{,}970 \text{ pounds, compression in } bC$$

Likewise,

$$\Sigma X = S_2 + S_3 \cos H + S_5 \cos H - S_6 = 0;$$

whence

$$S_6 = S_2 + S_3 \cos H + S_5 \cos H$$
$$= 11{,}250 \cot H + 11{,}250 \csc H \cos H + 6{,}750 \csc H \cos H$$
$$= 29{,}250 \cot H = 18{,}280 \text{ pounds, tension in } bc$$

Joint C.—This joint is represented in Fig. 6 (C), the forces acting on it being S_4, S_5, S_7, and S_8, which represent the stresses in BC, bC, Cc, and CD, respectively; S_4 acts horizontally to the right, S_5 upwards toward the right, while S_7 and S_8 are unknown. The stress in Cc will be assumed as tension, and that in CD as compression. Then, S_7 will act downwards to the right, and S_8 horizontally to the left.

$$\Sigma Y = S_5 \sin H - S_7 \sin H = 0;$$

whence

$$S_7 \sin H = S_5 \sin H = 6{,}750 \text{ pounds}$$

and

$$S_7 = 6{,}750 \csc H = 7{,}970 \text{ pounds, tension in } Cc$$

Likewise,

$$\Sigma X = S_4 + S_5 \cos H + S_7 \cos H - S_8 = 0;$$

whence

$$S_8 = S_4 + S_5 \cos H + S_7 \cos H$$
$$= 22{,}500 \cot H + 6{,}750 \csc H \cos H + 6{,}750 \csc H \cos H$$
$$= 36{,}000 \cot H = 22{,}500 \text{ pounds, compression in } CD$$

Joint c.—This joint is represented in Fig. 6 (c), the forces acting on it being S_6, S_7, S_9, and S_{10}, which represent the

stresses in *bc*, *Cc*, *cD*, and *cd*, respectively; S_6 acts horizontally to the left, and S_7 upwards to the left; while S_9 and S_{10} are unknown. The stress in *cD* will be assumed as compression, and that in *cd* as tension. Then, S_9 will act downwards to the left, and S_{10} horizontally to the right.

$$\Sigma Y = S_7 \sin H - S_9 \sin H - 4{,}500 = 0;$$

whence

$$S_9 \sin H = S_7 \sin H - 4{,}500 = 6{,}750 - 4{,}500 = 2{,}250 \text{ pounds}$$

and

$$S_9 = 2{,}250 \csc H = 2{,}660 \text{ pounds, compression in } cD$$

Likewise,

$$\Sigma X = S_6 + S_7 \cos H + S_9 \cos H - S_{10} = 0;$$

whence

$$\begin{aligned} S_{10} &= S_6 + S_7 \cos H + S_9 \cos H \\ &= 29{,}250 \cot H + 6{,}750 \csc H \cos H + 2{,}250 \csc H \cos H \\ &= 38{,}250 \cot H = 23{,}910 \text{ pounds, tension in } cd \end{aligned}$$

Joint D.—This joint is represented in Fig. 6 (*D*), the forces acting on it being S_8, S_9, S_{11}, and S_{12}, which represent the stresses in *CD*, *cD*, *Dd*, and *DD'*, respectively; S_8 acts horizontally to the right, and S_9 upwards to the right; while S_{11} and S_{12} are unknown. The stress in *Dd* will be assumed as tension, and that in *DD'* as compression. Then, S_{11} will act downwards to the right, and S_{12} horizontally to the left.

$$\Sigma Y = S_9 \sin H - S_{11} \sin H = 0;$$

whence

$$S_{11} \sin H = S_9 \sin H = 2{,}250 \text{ pounds}$$

and

$$S_{11} = 2{,}250 \csc H = 2{,}660 \text{ pounds, tension in } Dd$$

Likewise,

$$\Sigma X = S_8 + S_9 \cos H + S_{11} \cos H - S_{12} = 0;$$

whence

$$\begin{aligned} S_{12} &= S_8 + S_9 \cos H + S_{11} \cos H \\ &= 36{,}000 \cot H + 2{,}250 \csc H \cos H + 2{,}250 \csc H \cot H \\ &= 40{,}500 \cot H = 25{,}310 \text{ pounds, compression in } DD' \end{aligned}$$

Joint d.—This joint is represented in Fig. 6 (*d*). It is evident at once that S_{11} is numerically equal to S_{13}, and that S_{10} is numerically equal to S_{14}. Therefore, the stress in *dD'* is 2,660 pounds tension, and that in *dc'* is 23,910 pounds tension

It is unnecessary to proceed further than joint *d*, as the stresses in the members at the right end are the same as those in the corresponding members at the left end.

From the preceding discussion, it may be seen that the stress in any web member is equal to the algebraic sum of all the vertical forces that act on the truss on the left of the member considered, that is, to the shear in the panel in which the member is located, multiplied by $\csc H$ (see Art. **10**).

17. For the chord members, it is convenient to refer again to the stress in one of the members, such as $c\,d$, Fig. 6 (*c*) (*joint c*). The equation $\Sigma X = 0$ gives

$$S_{10} = S_6 + S_7 \cos H + S_9 \cos H$$

Substituting for S_6 its value $S_2 + S_3 \cos H + S_5 \cos H$,

$$S_{10} = S_2 + S_3 \cos H + S_5 \cos H + S_7 \cos H + S_9 \cos H$$

Likewise, substituting for S_2 its value $S_1 \cos H$,

$$S_{10} = S_1 \cos H + S_3 \cos H + S_5 \cos H + S_7 \cos H + S_9 \cos H$$

Letting Y_1, Y_3, etc. represent the vertical components of the stresses S_1, S_3, etc., and substituting for S_1, S_3, etc. their values $Y_1 \csc H$, $Y_3 \csc H$, etc., respectively, we have

$$\begin{aligned} S_{10} &= Y_1 \csc H \cos H + Y_3 \csc H \cos H + Y_5 \csc H \cos H \\ &\qquad + Y_7 \csc H \cos H + Y_9 \csc H \cos H \\ &= Y_1 \cot H + Y_3 \cot H + Y_5 \cot H + Y_7 \cot H \\ &\qquad + Y_9 \cot H = (Y_1 + Y_3 + Y_5 + Y_7 + Y_9) \cot H \end{aligned}$$

Now, $Y_1 \cot H$, $Y_3 \cot H$, etc. are the horizontal components of the stresses in $a\,B$, $B\,b$, etc., respectively, and the sum of these components from $Y_1 \cot H$ to $Y_9 \cot H$ is the algebraic sum of the horizontal components of the stresses in all the web members that connect with the lower chord at the left of $c\,d$. In like manner, it may be shown that the stress in $D\,D'$ is equal to the algebraic sum of the horizontal components of the stresses in all the web members that connect with the upper chord to the left of $D\,D'$. In general,

The stress in any portion of either chord is equal to the algebraic sum of the horizontal components of the stresses in all the web members that connect with the chord at the left of the portion considered.

18. In the present case, the web members all make the same angle with the horizontal; that is, H is constant, and the stress S_{10} in a chord member, such as $c\,d$, is equal to $(Y_1 + Y_3 + Y_5 + Y_7 + Y_9) \cot H$. Letting ΣY represent the sum of the vertical components in all the web members that connect with a chord at the left of any portion, then, the stress in that portion may be found by the formula

$$S = \Sigma Y \times \cot H$$

In applying this formula to the determination of the stress in a chord member, care must be taken that the horizontal components are given the proper signs. For example, for $c\,d$, the truss may be considered cut by the section $p\,q$, Fig. 6 (e), and the portion below and to the left of this section treated as a free body. The horizontal forces that act on this portion are the horizontal components of S_1, S_3, etc., and the stress in $c\,d$: S_1, S_5, and S_9 act downwards to the left, and S_3 and S_7 act upwards to the left; therefore, all the horizontal components of these stresses act to the left, and in finding S_{10} the vertical components of the stresses from S_1 to S_9 must be *added* numerically to find ΣY.

From the foregoing, the following general rule is derived:

To find the stress in any web member of a single-system Warren truss by the method of joints, multiply the shear in the panel in which the member is located by csc H; to find the stress in any chord member, multiply by cot H the algebraic sum of all the shears used in obtaining the stresses in all the web members that connect with the chord at the left of the member considered.

The application of this rule can be greatly simplified by constructing a diagram, as shown in Fig. 7. A sketch of the truss is drawn (not necessarily to scale) and on the upper side of each web member is written, with its proper sign and as the coefficient of csc H, the shear in the panel in which the member is located. On the upper side of each chord member is written, as the coefficient of cot H, the algebraic sum of the shears that have been written on all the web members that connect with the chord at the left of the member considered. On the under side of each member is written the stress obtained by performing the indicated multiplication.

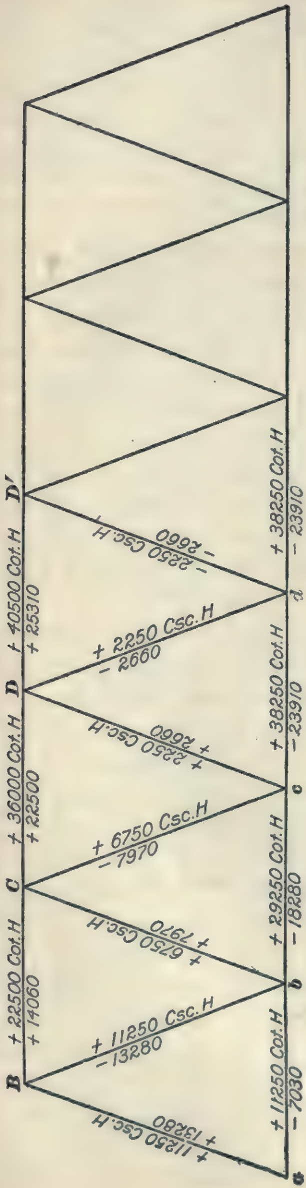

B
C
D
D'
a
b
c
d
+ 22500 Cot. H
+ 14060
+ 36000 Cot. H
+ 22500
+ 40500 Cot. H
+ 25310
+ 11250 Cot. H
− 7030
+ 29250 Cot. H
− 18280
+ 38250 Cot. H
− 23910
+ 38250 Cot. H
− 23910
+ 11250 Csc. H
+ 13280
+ 11250 Csc. H
− 13280
+ 6750 Csc. H
+ 7970
+ 6750 Csc. H
− 7970
+ 2250 Csc. H
+ 2660
+ 2250 Csc. H
− 2660
− 2250 Csc. H
− 2660

FIG. 7

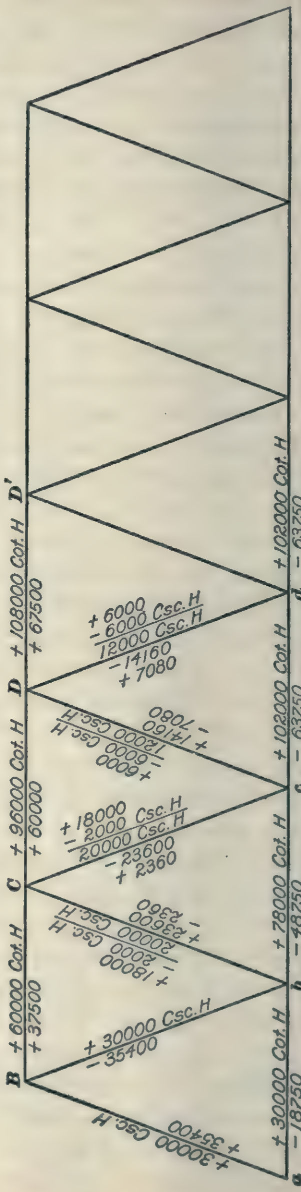

B
C
D
D'
a
b
c
d
+ 60000 Cot. H
+ 37500
+ 96000 Cot. H
+ 60000
+ 108000 Cot. H
+ 67500
+ 30000 Cot. H
− 18750
+ 78000 Cot. H
− 48750
+ 102000 Cot. H
− 63750
+ 102000 Cot. H
− 63750
+ 30000 Csc. H
+ 35400
+ 30000 Csc. H
− 35400
+ 18000
− 2000
20000 Csc. H
+ 23600
− 2360
+ 18000
− 2000 Csc. H
20000 Csc. H
− 23600
+ 2360
+ 6000
− 6000
12000 Csc. H
+ 14160
− 7080
+ 6000
− 6000 Csc. H
12000 Csc. H
− 14160
+ 7080

FIG. 8

The plus and minus signs written before the stresses indicate, as usual, compression and tension, respectively. Thus, in Fig. 7, the coefficient +6,750 of csc H on cC is the shear in the panel bc; the product 6,750 csc H gives 7,970, which is the numerical value of the stress in cC. The coefficient +29,250 of cot H on bc is the algebraic sum of the shears written on the members ab, Bb, and bC; the product 29,250 cot H gives 18,280, which is the numerical value of the stress in bc.

19. Live-Load Chord Stresses.—As the chord stresses are greatest when the truss is fully loaded, it is necessary first to find the shears due to a full live load. They are as follows:

Panel	Shear, in Pounds
ab	30,000
bc	18,000
cd	6,000

These values are written on the upper side of the web members, as shown in Fig. 8, and the coefficients of cot H for the chord stresses are found by adding the shears as explained in Art. **18**. Then, the stresses in the chord members are obtained by performing the multiplications indicated, and the results written on the under sides of the members.

20. Live-Load Web Stresses.—The shears that were found in Art. **19** are those due to full live load, the shear in any panel being the difference between the left reaction and the sum of all the panel loads between the left reaction and the panel under consideration. For example, the shear in panel cd due to full live load is

$$\frac{12{,}000\,(1+2+3+4+5)}{6}-(12{,}000+12{,}000)$$

or,

$$\left[\frac{12{,}000\,(1+2+3)}{6}\right]+\left[\frac{12{,}000\,(4+5)}{6}-(12{,}000+12{,}000)\right]$$

$$=\left[\frac{12{,}000\,(1+2+3)}{6}\right]-\left[\frac{12{,}000\,(1+2)}{6}\right]$$

The expression contained in the left-hand brackets of the last member of this equation is the left reaction that would be caused by loads at d, c', and b', if they were the only loads on the truss; it was explained in Art. **13** that this is the maximum positive live-load shear in panel $c\,d$. Also, the expression contained in the right-hand bracket is the maximum negative live-load shear in panel $c\,d$. From this the following principle is obtained:

The shear in any panel of a truss due to full live load is equal to the algebraic sum of the maximum live-load positive shear and the maximum live-load negative shear that can occur in that panel.

Let V'' = shear in any panel due to full live load;

V_p'' = maximum positive live-load shear that can occur in that panel;

V_n'' = maximum negative live-load shear that can occur;

then, $$V'' = V_p'' + V_n'';$$
whence $$V_p'' = V'' - V_n'';$$
that is, the maximum positive live-load shear in any panel may be found by subtracting algebraically the maximum negative live-load shear that can occur in the panel from the shear due to full live load. This principle is of special value in finding live-load web stresses by the method of joints; the shear in each panel due to full load is found in connection with the chord stresses; the maximum negative shear in each panel is then found in order to get the minimum stresses in the members; then, the maximum positive shear in any panel may be found by subtracting algebraically the maximum negative shear from the shear in the panel due to full load.

In panel $a\,b$ there can be no negative shear. Then, in this panel the maximum live-load stresses occur when the truss is fully loaded; csc H may be written after the shear that has been written on $a\,B$ and $B\,b$ (30,000 pounds), Fig. 8, and the stresses found by multiplying that shear by csc H (1.18). The results are written on the other side of the lines that represent $a\,B$ and $B\,b$. On the other web members,

directly under the values of the shear due to full load, are written, as coefficients of csc H, the maximum negative live-load shear and the maximum positive live-load shear, the latter being obtained by subtracting, algebraically, the negative shear from the shear due to full load. The maximum and minimum stresses are obtained by performing the multiplications indicated, and the results are written on the under side of the members. As stated in Art. **18**, the plus and minus signs written before the stresses represent compression and tension, respectively. Thus, in panel bc, Fig. 8, the shear due to full load (+18,000 pounds) is written on bC and Cc; the maximum negative shear (−2,000 pounds) is written under +18,000, and is the coefficient of csc H for the minimum live-load stresses in bC and Cc. The algebraic difference between the shear due to full load and the maximum negative shear, $+18{,}000-(-2{,}000) = +20{,}000$ pounds, is then written under −2,000 csc H, as the coefficient of csc H, for the maximum live-load stresses in bC and Cc. These stresses are obtained by performing the multiplications, and their values are written on the under sides of the members. Thus, for the member bC, the maximum live-load stress is 20,000 csc H, or +23,600 pounds; the minimum live-load stress is −2,000 csc H, or −2,360 pounds.

From the foregoing, the following general rule is obtained: *To find the maximum and minimum live-load stresses in the web members of a single-system Warren truss, when the shears due to full live load are known, write on each member the maximum negative live-load shear in the panel in which the member is located, and multiply it by csc H for the minimum stress; subtract, algebraically, the maximum negative shear from that due to full load, and multiply the result for each member by csc H for the maximum stresses.*

The combined stresses are found in the same way as in Art. **14**.

EXAMPLE.—The truss represented in Fig. 9 is a seven-panel through Warren truss, with dimensions as shown. The dead load is 1,000 pounds, and the live load, 2,000 pounds per linear foot of bridge. Assuming that one-third of a dead panel load is applied at each of the

joints of the upper chord, and two-thirds at the joints of the lower chord, find, by the method of joints, the maximum and minimum stresses in all the members.

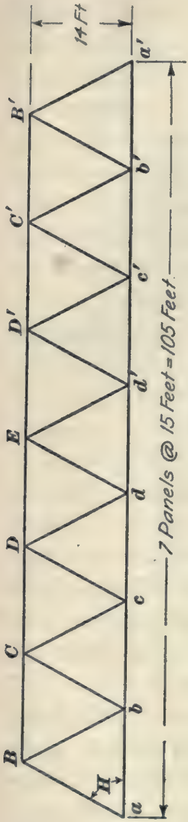


FIG. 9

SOLUTION.—Each dead panel load is equal to

$$\frac{1{,}000 \times 15}{2} = 7{,}500 \text{ pounds}$$

of which 5,000 pounds is applied at the joints of the lower chord, and 2,500 pounds at the joints of the upper chord. It will be noticed that there are six joints in the lower chord and seven in the upper chord. It is customary to assume that one-third of a panel load is applied at each of the joints of the upper chord. The dead-load reaction for one truss is equal to

$$\frac{5{,}000 \times 6}{2} + \frac{2{,}500 \times 7}{2} = 23{,}750 \text{ pounds}$$

Each live panel load is equal to

$$\frac{2{,}000 \times 15}{2} = 15{,}000 \text{ pounds}$$

and the live-load reaction for one truss fully loaded is equal to

$$\frac{15{,}000 \times 6}{2} = 45{,}000 \text{ pounds}$$

As a portion of the dead load is applied at the upper-chord joints, which lie midway between the joints of the lower chord, the dead-load shear in any panel of the lower chord is not constant. For example, in the panel $b\,c$, the dead-load shear from b to C is equal to

$$23{,}750 - (5{,}000 + 2{,}500) = 16{,}250 \text{ pounds}$$

while from C to c it is

$$23{,}750 - (5{,}000 + 2{,}500 + 2{,}500), \text{ or } 13{,}750 \text{ pounds}$$

The figure gives

$$\csc H = \frac{\sqrt{14^2 + 7.5^2}}{14} = 1.134; \ \cot H = \frac{7.5}{14} = .5357$$

The dead-load stresses, found by the rule given in Art. **18**, are indicated in Fig. 10 (a); the live-load stresses, found by the rule given in Art. **20**, are indicated in Fig. 10 (b). As the dead-load and live-load stresses are not required separately, the work will be shortened in the present case by combining the coefficients of $\csc H$ and $\cot H$, respectively, and indicating the maximum and minimum stresses, as represented in Fig. 10 (c). There remains now simply the operation of multiplying these coefficients by $\csc H$ and $\cot H$, respectively, to get the maximum and the minimum combined stresses, as represented in Fig. 10 (d). The student should verify the values of these

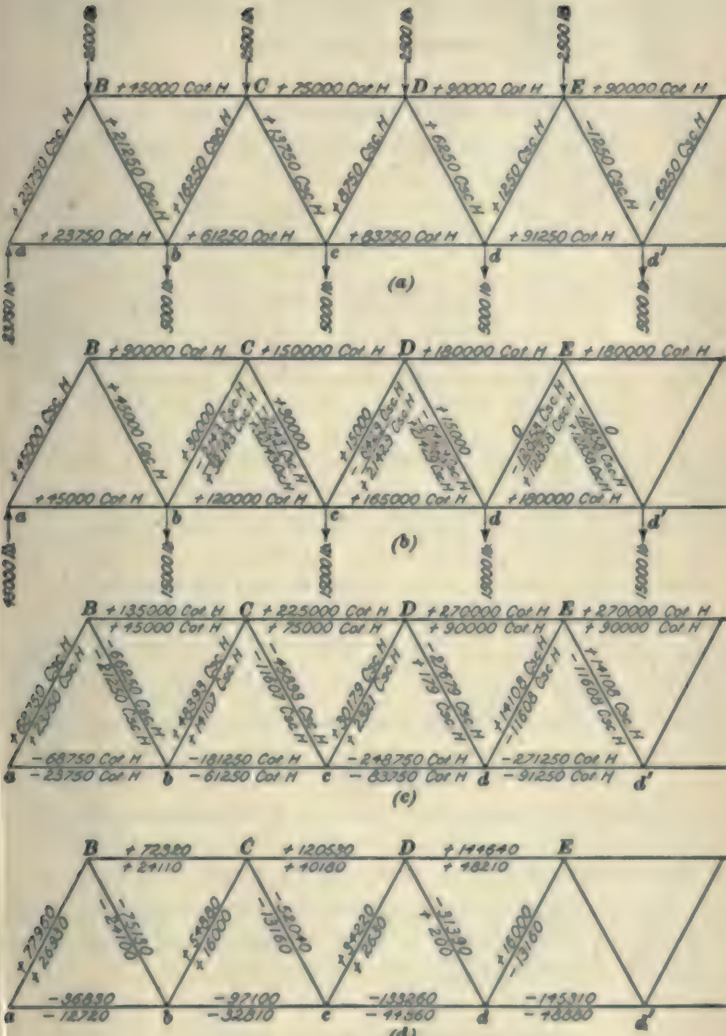

B
C
D
E
+45000 Cot H
+75000 Cot H
+90000 Cot H
+90000 Cot H
+23750 Cot H
+61250 Cot H
+83750 Cot H
+91250 Cot H
a
b
c
d
d′
(a)
+90000 Cot H
+150000 Cot H
+180000 Cot H
+180000 Cot H
+45000 Cot H
+120000 Cot H
+165000 Cot H
+180000 Cot H
(b)
+135000 Cot H
+45000 Cot H
+225000 Cot H
+75000 Cot H
+270000 Cot H
+90000 Cot H
+270000 Cot H
+90000 Cot H
−68750 Cot H
−23750 Cot H
−181250 Cot H
−61250 Cot H
−248750 Cot H
−83750 Cot H
−271250 Cot H
−91250 Cot H
(c)
+72320
+24110
+120530
+40180
+144640
+48210
−36830
−12720
−97100
−32810
−133260
−44360
−145310
(d)

FIG. 10

stresses. The signs of the coefficients given in Figs. 10 (*a*) and (*b*) are the signs of the shears; the minus and plus signs in Fig. 10 (*c*) and (*d*) represent tension and compression, respectively.

THE DECK WARREN TRUSS

21. When used in a deck bridge, the Warren truss may be supported either as shown in Fig. 11 or as shown in Fig. 12. The live load is supported at the joints of the upper chord; the dead load may be assumed to be applied at

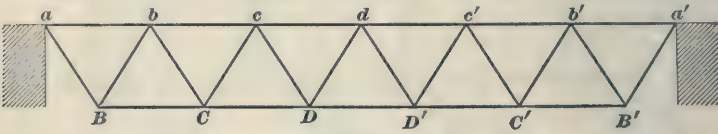


FIG. 11

the joints of the upper chord, or one-third of it at the joints of the lower chord. In calculating the stresses, the same methods and rules are used as for the through truss.

In Fig. 11, each panel load is a full load. In Fig. 12, the loads are supported between A and B, and between B' and A',

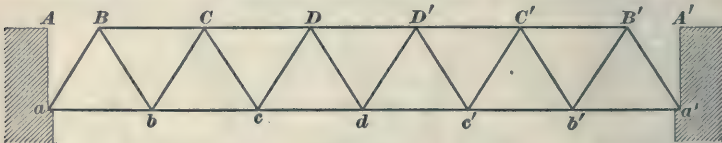


FIG. 12

by the end stringers, one end of which rests on the abutments, and the other end connects with the floorbeam at B or B'. As the distances AB and $B'A'$ are each equal to a half panel, each of the joints B and B' supports three-quarters of a panel load, and this value must be used at these joints in the calculation of reactions and stresses.

EXAMPLES FOR PRACTICE

1. Suppose that the truss represented in Fig. 9 has a span of 112 feet, and a height of 16 feet; if the dead load is equal to 800 pounds, all of which is applied at the joints of the loaded chord, and the live load is 1,800 pounds per linear foot of bridge, find: (*a*) the maximum and minimum combined stresses in the members bc, CD, and dd', using

the method of joints: (*b*) the maximum and minimum combined stresses in the members *a B*, *b C*, and *d D*, using the method of sections.

Ans.

	Member	Stress, in Pounds: Maximum	Stress, in Pounds: Minimum
(*a*)	*b c*	− 83,200	− 25,600
	C D	+ 104,000	+ 32,000
	d d'	− 124,800	− 38,400
(*b*)	*a B*	+ 69,800	+ 21,500
	b C	+ 48,900	+ 12,000
	d D	− 30,200	− 250

2. Let Fig. 9 represent a seven-panel deck Warren truss having the same loads and dimensions as in example 1 and supported in a manner similar to that shown in Fig. 12. What are the maximum and minimum stresses due to combined dead and live load: (*a*) in the members *b c*, *C D*, and *d d'*, using the method of sections? (*b*) in the members *B b*, *b C*, and *D d*, using the method of joints?

Ans.

	Member	Stress, in Pounds: Maximum	Stress, in Pounds: Minimum
(*a*)	*b c*	− 85,800	− 26,400
	C D	+ 101,400	+ 31,200
	d d'	− 127,400	− 39,200
(*b*)	*B b*	− 59,000	− 17,000
	b C	+ 59,000	+ 17,000
	D d	− 21,700	+ 6,500

THE WARREN TRUSS WITH SUBVERTICALS

22. Description.—The simple type of Warren truss can be used for span lengths up to about 125 feet. For longer spans, it is impossible to fulfil the economical conditions of height, panel length, and slope of diagonals. If the proper height of truss is used and the diagonals are given an economical inclination, the panels will be too long, and it is advisable to subdivide them. This may be accomplished in several ways, one of which is to use a Warren truss with vertical members attached to the joints of the unloaded chord, dividing each panel of the loaded chord into two equal panels. The truss is then called the **Warren truss with subverticals.** The vertical members are tension members in a through truss and compression members in a deck truss. All the other members correspond in every way to those in the through Warren truss in Fig. 1. The method of calculation is the same as for the single-system Warren truss.

In Fig. 13 (a) is represented a twelve-panel through Warren truss with subverticals, having a span of 180 feet and a height of 24 feet. Each panel load will be denoted by W, and the reactions, as usual, by R_1 and R_2.

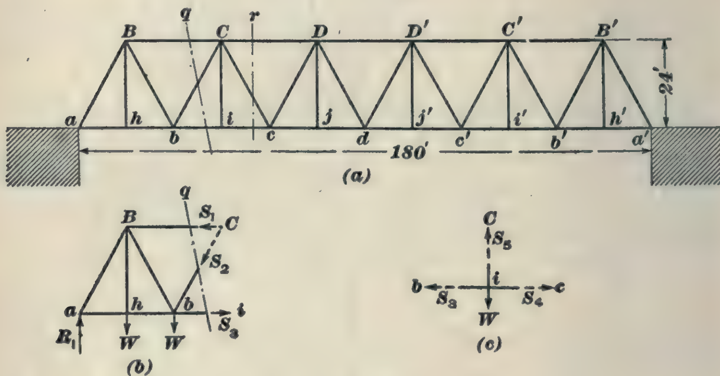


FIG. 13

23. Chord Stresses.—The stresses in the upper-chord members may be found by dividing the bending moments at the opposite joints b, c, d, etc. by the height of the truss; the stresses in the lower-chord members may be found by means of the bending moments at B, C, D, etc. (see Art. 5). Thus, for the stresses in $B\,C$ and $b\,i$, the truss may be considered cut by a plane q. The portion to the left of section q is shown in Fig. 13 (b), S_1, S_2, and S_3 representing the stresses in the members $B\,C$, $b\,C$, and $b\,i$, respectively. The stress in $B\,C$ is compression, and so S_1 will act horizontally to the left; the stress in $b\,i$ is tension, and so S_3 will act horizontally to the right. For the stress in $B\,C$, the center of moments is taken at b. Then,

$$\Sigma M = R_1 \times 30 - W \times 15 - S_1 \times 24 = 0;$$

whence

$$S_1 = \frac{R_1 \times 30 - W \times 15}{24} = \frac{\text{bending moment at } b}{24}$$

For the stress in $b\,i$, the center of moments is taken at C. Then,

$$\Sigma M = R_1 \times 45 - W \times 30 - W \times 15 - S_3 \times 24 = 0;$$

whence

$$S_s = \frac{R_1 \times 45 - W \times 30 - W \times 15}{24} = \frac{\text{bending moment at } C}{24}$$

For the stress in $i\,c$, the joint i is treated as a free body, as shown in Fig. 13 (c). The only horizontal forces are S_s and S_s; therefore, they are equal and opposite, and the stress in $i\,c$ is equal to the stress in $b\,i$. In like manner, the stress in $a\,h$ is equal to the stress in $h\,b$; the stress in $c\,j$ is equal to the stress in $j\,d$, etc. Other chord stresses may be determined in the same way as those here explained.

24. Web Stresses.—The stress in each vertical is tension and equal to the load applied at the foot of the vertical. This is evident when the equation $\Sigma Y = 0$ is applied to the forces acting on such a joint as i, Fig. 13 (c). The only vertical forces being S_s and the panel load W, they must be equal and opposite. Therefore, the stress in each vertical is equal to a panel load. The other web stresses may be found by the method of shears already explained (Art. 10).

25. Deck Bridge.—If the through truss in Fig. 13 is inverted and used as a deck truss, as shown in Fig. 14, the maximum stresses in members having the same letters in

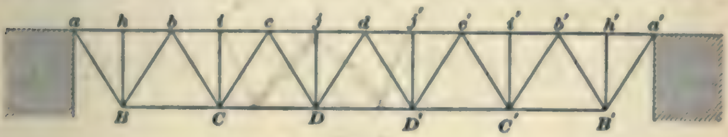


FIG. 14

the two figures will be numerically equal, but of opposite characters. If the truss is supported as shown in Fig. 15, the stresses in all the members but the verticals will be of the

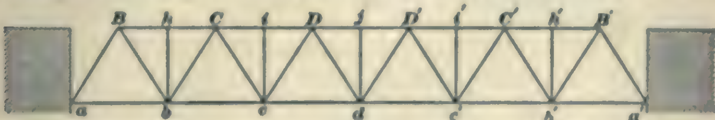


FIG. 15

same characters and have the same numerical values as those in the corresponding members in the through truss in Fig. 13. The verticals will be in compression.

EXAMPLES FOR PRACTICE

1. Suppose that, in the bridge shown in Fig. 13, the dead load is 1,000 pounds, and the live load, 2,200 pounds, per linear foot. Assume that all the dead load is applied at the joints of the loaded chord. What are the maximum and minimum stresses, due to the combined dead and live load, in all the members?

Ans.

Member	Stress, in Pounds Maximum	Stress, in Pounds Minimum
a B, *a′ B′*	+ 155,700	+ 48,600
B C, *B′ C′*	+ 150,000	+ 46,900
C D, *C′ D′*	+ 240,000	+ 75,000
D D′	+ 270,000	+ 84,400
a h, *h b*, *a′ h′*, *h′ b′*	− 82,500	− 25,800
b i, *i c*, *b′ i′*, *i′ c′*	− 202,500	− 63,300
c j, *j d*, *c′ j′*, *j′ d*	− 262,500	− 82,000
B h, *C i*, *D j*, *D′ j′*, *C′ i′*, *B′ h′*	− 24,000	− 7,500
B b, *B′ b′*,	− 129,000	− 38,200
b C, *b′ C′*	+ 104,000	+ 26,000
C c, *C′ c′*	− 80,500	− 12,400
c D, *c′ D′*	+ 58,700	− 2,900
D d, *D′ d*	− 38,500	+ 19,900

2. Let Fig. 16 be a ten-panel deck bridge having a span length of 150 feet and a height of 20 feet. If the dead load is 900 pounds, and the live load, 2,000 pounds, per linear foot, and it is assumed that all

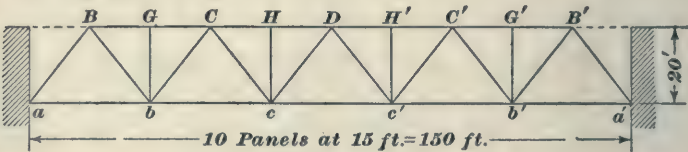


FIG. 16

the dead load is applied at the joints of the loaded chord, find: (*a*) the maximum and minimum stresses in the members *a b*, *B b*, *G b*, and *b C* due to combined dead and live load; (*b*) the maximum and minimum stresses in the members *b c*, *H D*, *c D*, and *c c′* due to combined dead and live load.

Ans.

	Member	Stress, in Pounds Maximum	Stress, in Pounds Minimum
(*a*)	*a b*	− 73,400	− 22,800
	B b	− 97,000	− 27,700
	G b	+ 21,800	+ 6,800
	b C	+ 73,600	+ 15,500
(*b*)	*b c*	− 171,300	− 53,200
	H D	+ 195,800	+ 60,800
	c D	+ 32,300	− 14,500
	c c′	− 203,900	− 63,300

THE DOUBLE-INTERSECTION WARREN TRUSS

26. Description.—Fig. 17 (*a*) shows another type of Warren truss with subdivided panels, which was extensively used in the past and is used to some extent at the present time. The simple Warren truss is shown in full lines, the panels being subdivided by the addition of the web members shown in dotted lines parallel to the full-line members and

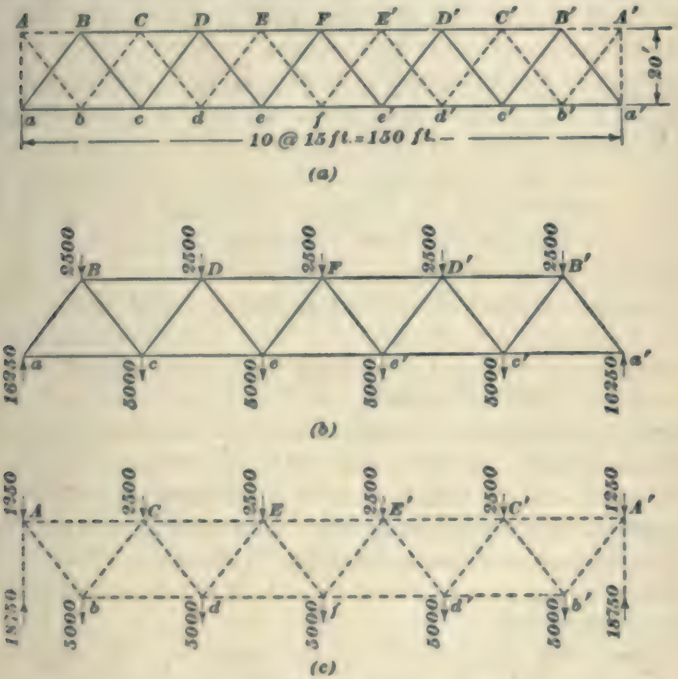


FIG. 17

half way between them. The two sets of web members are called the *two systems of web*. Such a truss is called a **double system, or double intersection, Warren truss**, and sometimes simply a **double Warren truss**. The joints of one system are in each case vertically opposite the joints of the other. As the end diagonals of the dotted system slope upwards, it is necessary to provide a vertical member, called **the vertical end post**, and produce the top chord at each end.

27. Methods of Calculation.—If the truss shown in Fig. 17 (*a*) is considered cut by a plane that intersects two chord members, that plane will cut also two web members, and there will be four unknown stresses. As there are only three equations of equilibrium, they are not sufficient for the determination of the four unknown stresses unless some assumption is made regarding the distribution of stress among the several members cut by the plane. It is customary to assume that the two systems of web members act independently, or, in other words, that they are the web members of two independent trusses lying in the same plane, the top and bottom chords being common to both trusses. The stresses in the web members of each system may be found from the loads that come on the system; and the chord stresses may be found by properly combining the chord stresses of the two systems. This will be made clearer by studying Figs. 17 (*b*) and (*c*). The system shown as a truss in Fig. 17 (*b*) is assumed to support the loads at $c, e, e', c', B, D, F, D'$, and B'. The web stresses due to these loads are the actual web stresses in the corresponding members of the truss shown in Fig. 17 (*a*); and the chord stresses are partial or component chord stresses. The system shown as a truss in Fig. 17 (*c*) is assumed to support the loads at $b, d, f, d', b', A, C, E, E', C'$, and A'. The web stresses due to these loads are the actual web stresses; the chord stresses are component chord stresses. The actual chord stresses may be found by adding the stresses found in the two systems.

The double-intersection Warren truss may be used in a deck or in a through bridge. The stresses are calculated in the same way for the two kinds. As the analytic method of calculation is shorter than the graphic, the latter will not be considered. The method of calculation can best be illustrated by an example. For this purpose, the dead-load stresses in the truss shown in Fig. 17 (*a*) will be determined. The truss has ten panels, the span length is 150 feet, and the height 20 feet. The dead load will be taken as 1,000 pounds per linear foot of bridge, one-third of which

is supposed to be applied at the unloaded chord. The method of joints is best adapted to this case.

28. Panel Loads and Reactions.—The truss may be divided into the two systems shown in Fig. 17 (*b*) and (*c*). For convenience of reference, the system shown in full lines in Fig. 17 (*b*) may be called the *primary system*, and that shown in dotted lines in Fig. 17 (*c*), the *secondary system*. The dead panel load is equal to

$$\frac{1{,}000 \times 15}{2} = 7{,}500 \text{ pounds}$$

of which 2,500 is supported at each of the top joints, and 5,000 at each of the bottom joints. The primary system supports four loads of 5,000 and five loads of 2,500 pounds. Therefore, the reaction for the primary system is

$$\frac{4 \times 5{,}000 + 5 \times 2{,}500}{2} = 16{,}250 \text{ pounds}$$

The secondary system supports five loads of 5,000 and four loads of 2,500 pounds. Therefore, the reaction for the secondary system is

$$\frac{5 \times 5{,}000 + 4 \times 2{,}500}{2} = 17{,}500 \text{ pounds}$$

In addition to this, there is a half-panel load of 1,250 pounds at each of the end joints of the top chord. Then, the total reaction for the secondary system is equal to 18,750 pounds. The loads and reactions for the primary systems are shown in Fig. 17 (*b*); those for the secondary system, in Fig. 17 (*c*). As in previous cases,

$$\cot H = \cot B\,a\,b = \frac{15}{20} = .75; \ \csc H = \frac{\sqrt{20^2 + 15^2}}{20} = 1.25$$

29. Web Stresses.—The stress in the vertical end post is equal to the reaction of the secondary system. The vertical components of the web stresses in each system may be written directly by finding the shears, and the stresses found from them by multiplying by $\csc H$. It should be borne in mind that each system is treated as an independent truss loaded as shown in Figs. 17 (*b*) and (*c*); also, that, in determining the shear on any section, both the lower- and the upper-chord loads should be taken into account. Thus, the shear

on a plane cutting DF, De, and ce, Fig. 17 (b), is 16,250 − (2,500 + 5,000 + 2,500), or the algebraic sum of the external forces acting at a, B, c, and D.

The web stresses, whose values should be verified by the student, are:

MEMBER	STRESS, IN POUNDS
aA	+ 18,750
aB	16,250 × 1.25 = + 20,300
Ab	17,500 × 1.25 = − 21,900
bC	12,500 × 1.25 = + 15,600
Bc	13,750 × 1.25 = − 17,200
cD	8,750 × 1.25 = + 10,900
Cd	10,000 × 1.25 = − 12,500
dE	5,000 × 1.25 = + 6,250
De	6,250 × 1.25 = − 7,800
eF	1,250 × 1.25 = + 1,600
Ef	2,500 × 1.25 = − 3,100

30. Chord Stresses.—As explained in Art. **17**, the stress in any chord member of a single-system Warren truss is equal to the algebraic sum of the horizontal components of the stresses in all the web members that connect with the chord on the left (or right) of the member in question. For example, the stress in DF, Fig. 17 (b), is equal to the sum of the horizontal components in aB, Bc, cD, and De; the stress in EE', Fig. 17 (c), is equal to the sum of the horizontal components in Ab, bC, Cd, dE, and Ef. The stress in EF, Fig. 17 (a), equals the sum of the stresses in DF, Fig. 17 (b), and EE', Fig. 17 (c). Therefore, the stress in EF equals the sum of the horizontal components in Ab, aB, Bc, bC, Cd, cD, De, dE, and Ef. In general,

The stress in any chord member of a double Warren truss is equal to the algebraic sum of the horizontal components of the stresses in all the web members that connect with the chord on the left (or right) of the member considered.

Keeping in mind that the horizontal component of the stress in any web member is equal to the vertical component multiplied by cot H, the chord stresses may be written as follows:

MEMBER	STRESS, IN POUNDS
AB	$17{,}500 \times .75 = +13\,100$
BC	$(17{,}500 + 16{,}250 + 13{,}750) \times .75 = +35{,}600$
CD	$(17{,}500 + 16{,}250 + 13{,}750 + 12{,}500 + 10{,}000) \times .75 = +52{,}500$
DE	$(17{,}500 + 16{,}250 + 13{,}750 + 12{,}500 + 10{,}000 + 8{,}750 + 6{,}250) \times .75 = +63{,}750$
EF	$(17{,}500 + 16{,}250 + 13{,}750 + 12{,}500 + 10{,}000 + 8{,}750 + 6{,}250 + 5{,}000 + 2{,}500) \times .75 = +69{,}400$
FE'	$(17{,}500 + 16{,}250 + 13{,}750 + 12{,}500 + 10{,}000 + 8{,}750 + 6{,}250 + 5{,}000 + 2{,}500 + 1{,}250 - 1{,}250) \times .75 = +69{,}400$
ab	$16{,}250 \times .75 = -12{,}200$
bc	$(16{,}250 + 17{,}500 + 12{,}500) \times .75 = -34{,}700$
cd	$(46{,}250 + 13{,}750 + 8{,}750) \times .75 = -51{,}600$
de	$(68{,}750 + 10{,}000 + 5{,}000) \times .75 = -62{,}800$
ef	$(83{,}750 + 6{,}250 + 1{,}250) \times .75 = -68{,}400$
fe'	$(91{,}250 + 2{,}500 - 2{,}500) \times .75 = -68{,}400$

31. Live-Load Stresses.—The live-load stresses may be found in precisely the same way as the dead-load stresses, by separating the truss into two systems. For the maximum chord stresses, each system should be fully loaded, and the stresses in the members added together to get the combined or actual stresses. For the maximum web stresses, the portion of each system that will give the maximum shear (positive or negative) in the various panels must be loaded; the stresses found from the shears will be the actual maximum and minimum live-load stresses in the web members.

EXAMPLE FOR PRACTICE

If the live load on the bridge described in Art. **26** and illustrated in Fig. 17 is 2,200 pounds per linear foot, determine: (*a*) the maximum and minimum combined stresses in the members EF, Ef, eF, and ef, using the dead-load stresses found in the preceding pages; (*b*) the maximum and minimum combined stresses in the members BC, Bc, bC, and bc.

Ans.

	Member	Stress, in Pounds: Maximum	Stress, in Pounds: Minimum
(*a*)	EF	+ 224,100	+ 69,400
	Ef	− 21,700	+ 5,100
	eF	+ 13,900	− 10,800
	ef	− 216,900	− 68,400
(*b*)	BC	+ 116,000	+ 35,600
	Bc	− 58,400	− 17,200
	bC	+ 48,600	+ 13,600
	bc	− 109,000	− 34,700

THE DOUBLE WARREN TRUSS WITH SUBVERTICALS

32. Description.—Fig. 18 (*a*) shows the double Warren truss with subverticals that subdivide each panel of the loaded chord into two equal panels. In this truss, the loads at the intermediate joints *b*, *d*, *f*, etc. act on both systems at the intersections *B*, *D*, *F*, etc. of the web members, and on this account it is impossible to separate the truss into two

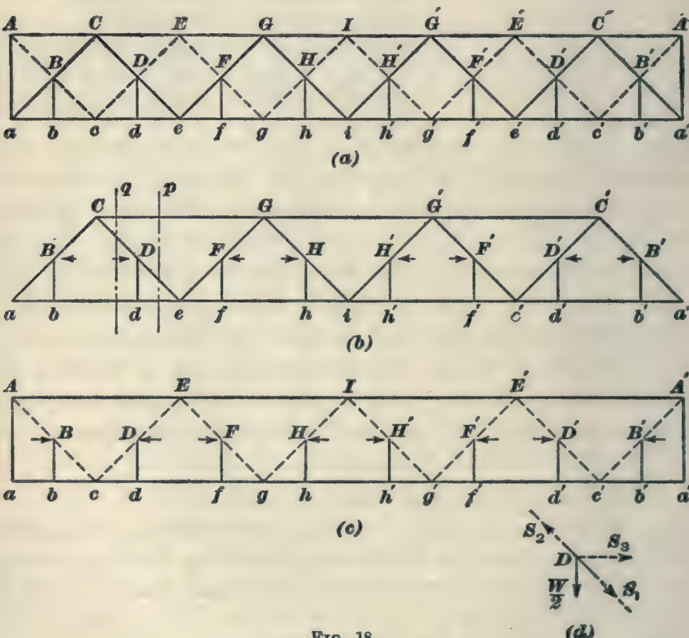


FIG. 18

independent systems. However, an assumption is usually made that is probably very close to the actual distribution of stresses. As in the case of the truss described in the preceding articles, one of the two systems formed by the inclined members is called the primary; the other, the secondary.

33. Method of Calculation.—The stress in each subvertical is evidently tension, and equal to the panel load at

its lower joint. In finding the stresses in the other members, it is customary to assume that each system carries one-half of the load transmitted by the subverticals to the joints B, D, F, etc. Thus, in Figs. 18 (b) and (c), which shows the primary and secondary systems, together with the subverticals, each of the joints b, d, f, etc. is supposed to carry one-half of a panel load to each system. The stresses are found in almost the same manner as in the double Warren truss without subverticals, except that, in treating each system as an independent truss, external forces, representing the action of the other system on the system under consideration, must be introduced at the joints D, F, etc., as will be explained presently.

34. Web Stresses.—If the general equation of equilibrium $\Sigma Y = \Sigma S \sin H = 0$ is applied to all the external forces acting on one side of a plane of section that cuts a web member and two chord members of either system, such as section q, Fig. 18 (b), it will be seen that the vertical component of the stress in the web member is equal to the shear on the section, and the stress is equal to the shear multiplied by csc H. For the maximum or minimum stress in any member, the system in which the member occurs should be loaded on the right or left of the member, in the same way as in a single-system Warren truss.

Consider the sections q and p, Fig. 18 (b). Denoting the load at each lower-chord joint by W, the vertical component of the stress in CD is $R_1' - \frac{W}{2}$, and that in De is $R_1' - \frac{W}{2} - \frac{W}{2}$; then, the horizontal component of the stress in CD is equal to $\left(R_1' - \frac{W}{2}\right)\cot H$, and in De, to $\left(R_1' - \frac{W}{2} - \frac{W}{2}\right)\cot H$. Writing the expression for the sum of the horizontal forces at joint D of the primary system, shown in Fig. 18 (d), we have

$$\Sigma X = S_1 \cos H - S_1 \cos H$$

$$= \left(R_1' - \frac{W}{2}\right)\cot H - \left(R_1' - \frac{W}{2} - \frac{W}{2}\right)\cot H = \frac{W}{2}\cot H,$$

from which it will be seen that there is an unbalanced force,

equal to $\frac{W}{2}\cot H$, acting horizontally to the left, which must be held in equilibrium by the force S_3, equal to $\frac{W}{2}\cot H$, acting horizontally to the right. It may be shown that at the joint D of the secondary system there is also an unbalanced force equal to $\frac{W}{2}\cot H$, acting horizontally to the right, which holds in equilibrium the unbalanced force at joint D of the primary system. This force, which may be called S_3, is exerted at each joint (B, D, F, etc.) of each system by the other system, and may be considered as an external force in writing equations.

35. Chord Stresses.—The maximum chord stresses obtain when there is a full live load; the minimum, when there is no live load on the truss. The stress in any member may be found by properly combining the partial stresses in the two systems. When all the stresses are desired, the method of joints is the shorter; when only one or two stresses are desired, the method of moments is shorter.

For example, the stress in EG, Fig. 18 (a), is equal to the stress in CG, Fig. 18 (b), plus the stress in EI, Fig. 18 (c). By the method of joints, the stress in CG, Fig. 18 (b), is equal to the sum of the horizontal components of the stresses in BC and CD; the stress in EI is equal to the sum of the horizontal components of the stresses in AB, DE, and EF.

If it is desired to calculate the stress in any chord member by the method of moments, it is necessary to take into account the moments of the horizontal forces S_3. As these forces are alternately opposite in direction, it is convenient to pass the planes of section through the truss in such a way that there will be an even number of intermediate joints on the portion of the truss considered. Then, the moment of the forces S_3 on one side of the section about the center of moments will be zero (since there will be an even number whose resultant is zero), and they need not be considered in the equation of moments. In the present case, the planes

may be passed between a and b, d and f, h and h', etc. Thus, for the stress in ab, the truss may be cut by a vertical plane in panel ab, or in panel de, and the center of moments taken at B or C. With the center of moments at B, the stress in ab is

$$\frac{R_1' \times p}{\frac{h}{2}} = \frac{R_1 \times 2p}{h}$$

and, with the center of moments at C and section p, the stress in ab is, as before,

$$\frac{R_1 \times 2p - \frac{W}{2} \times p + \frac{W}{2} \times p}{h} = \frac{R_1 \times 2p}{h}$$

In this case, the load at D, being on the right of the center of moments, has a positive, or right-handed, moment, and similar cases must be carefully treated in order to get the signs correct. The stress in any chord member may be found in a manner similar to that just given.

EXAMPLE FOR PRACTICE

If the sixteen-panel through double Warren truss shown in Fig. 18 (a) has a span length of 192 feet and a height of 30 feet, and the dead load is 1,200 pounds per linear foot of bridge, all applied at the joints of the loaded chord, what are the dead-load stresses in the members CE, ef, GI, Ff, CD, eF, and HI?

Ans.

Member	Stress, in Pounds
CE	+ 57,600
ef	− 77,800
GI	+ 92,200
Ff	− 7,200
CD	− 27,600
eF	+ 13,800
HI	0

THE MULTIPLE-SYSTEM WARREN OR LATTICE TRUSS

36. Description.—When it is desirable to build a very deep truss of the Warren type, an economical inclination of diagonal and panel length may be used by adding two or three additional systems of web members to the simple type of truss, and subdividing the main panels into three or four

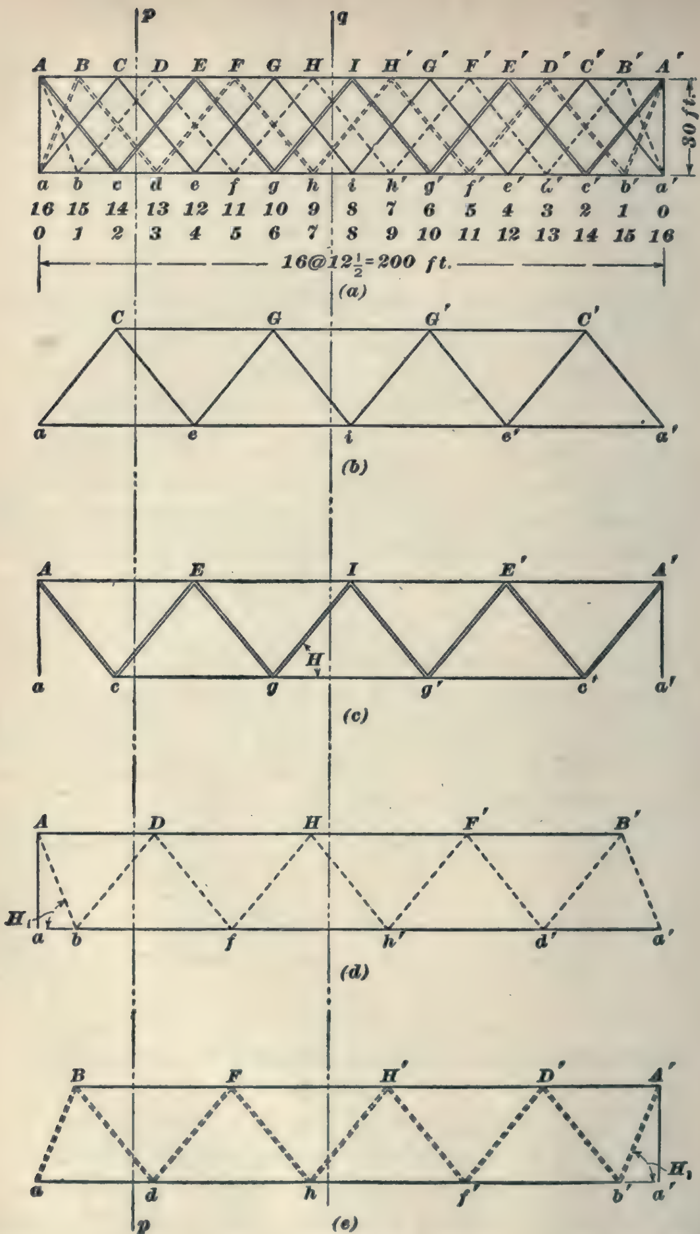

p
q
A B C D E F G H I H′ G′ F′ E′ D′ C′ B′ A′
30 ft.
a b c d e f g h i h′ g′ f′ e′ d′ c′ b′ a′
16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1 0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
16@12½=200 ft.
(a)
C G G′ C′
a e i e′ a′
(b)
A E I E′ A′
H
a c g g′ c′ a′
(c)
A D H F′ B′
H₁
a b f h′ d′ a′
(d)
B F H′ D′ A′
H₁
a d h f′ b′ a′
p
(e)

FIG. 19

equal panels. When there are four systems, as shown in Fig. 19 (*a*), the truss is called a **quadruple-system**, or **quadruple-intersection, Warren truss.** All trusses of this type are included under the general heading of "multiple-system" or "lattice trusses," and are usually built as riveted trusses. The web members of the different systems are riveted together at their intersections. Fig. 19 (*a*) represents a quadruple-intersection Warren truss with the four systems shown in single and double full and dotted lines. For convenience of reference, the system shown in single full lines in Fig. 19 (*b*) will be called the *primary system;* that shown in double full lines, Fig. 19 (*c*), the *secondary system;* that shown in single dotted lines, Fig. 19 (*d*), the *tertiary system;* and that shown in double dotted lines, Fig. 19 (*e*), the *quaternary system.*

37. Analysis of Stresses.—It is customary to assume that the only stresses in each system are those caused by the loads directly applied to it. Although this assumption is not strictly correct, it is probably as close as any assumption that can be made concerning the distribution of the stresses. The effect of connecting the web members to each other at every intersection is ignored in the calculation of stresses.

The stresses may be found in the same general way as the stresses in the double Warren truss, by dividing the truss into separate systems and calculating the stresses in the members of each system due to the loads that come on it. The stresses in those members (such as web members) that occur in only one system are the actual stresses; in those that occur in more than one system, they are component stresses, and the actual stresses are found by properly combining the component stresses in the different systems in which the members occur. The analytic method is best adapted to the determination of the stresses.

38. Stresses in Primary System.—Fig. 19 (*b*) shows the primary system, which is a single-intersection four-panel symmetrical Warren truss supported at the points *a* and *a'*.

The maximum and minimum stresses are found in the ordinary way; the stresses in the web members are the actual stresses in these members. The stresses in the chord members are component stresses.

39. Stresses in Secondary System.—Fig. 19 (*c*) shows the secondary system, which is a single-system symmetrical Warren truss supported at the points A and A' by the verticals $a\,A$ and $a'\,A'$. The maximum and minimum stresses are found in the ordinary way; the stresses in the members $a\,A$ and $a'\,A'$, and the stresses in the chord members are component stresses, as these members occur in more than one system; the stresses in the inclined web members are the actual stresses.

40. Stresses in Tertiary System.—Fig. 19 (*d*) shows the tertiary system, which is an unsymmetrical single-system Warren truss in which the end diagonals slope differently from the others. The truss is supported at the point A by the vertical $a\,A$, and at the point a' by the abutment. The maximum and minimum stresses are found in the same way as for the single-system Warren truss. The stress in $a\,A$ is equal to the left reaction. The stress in any chord member is equal to the bending moment due to the loads on the system, at the joint opposite the member, divided by the height of the truss. The stress in any web member is equal to the shear in the panel in which the member is located, multiplied by the cosecant of the angle that the member makes with the horizontal. As this system is unsymmetrical, it is necessary to find the dead-load stress and the maximum and minimum live-load stresses in every member, instead of finding them for only those members that are situated on one side of the center, as heretofore.

The maximum live-load chord stresses will occur when the truss is fully loaded; the maximum live-load stress in any web member due to positive shear, when all joints to the right of the member are loaded; the maximum live-load web stresses due to negative shear, when all joints to the left of the member are loaded.

41. Stresses in Quaternary System.—Fig. 19 (*e*) shows the quaternary system, which is the same as the tertiary system turned end for end. The remarks made in connection with the tertiary system apply here. The maximum and minimum stresses in any member of the quaternary system are equal to the stresses in the corresponding member at the other end of the tertiary system. Thus, the stress in $D' b'$ is equal to the stress in $D b$.

42. Stresses in End Members.—In calculating the actual stress in the end diagonals, such as $a B$, $A b$, etc., it should be remembered that the angle H_1, Fig. 19 (*d*) and (*e*), for these members is different from the angle H for the remaining diagonals, and that, therefore, the cosecant and cotangent will have different values for the two angles. If the stresses are found by the method of joints, it should be noted that $\cot H_1 = \frac{\cot H}{2}$, and that, then,

$$\text{hor. comp. in } a B = \text{vert. comp. in } a B \times \frac{\cot H}{2}$$

$$= \frac{\text{vert. comp. in } a B}{2} \times \cot H$$

This gives the value of the horizontal component in the end diagonal to be used in finding chord stresses by the method of joints, as explained in Art. **18**.

43. Actual Stresses.—The stresses found for the diagonals in the different systems are the actual stresses in the diagonals. The member $a A$ occurs in two systems, and the actual stress in it is equal to the sum of the stresses in $a A$, as found in the two systems. The chord member $A B$ occurs in two systems; $B C$, in three; $C D$, in four; etc.

44. Determination of Stresses by Method of Sections.—To illustrate the calculation of the stresses by the method of sections, let Fig. 19 (*a*) be a sixteen-panel through bridge having a span length of 200 feet and a height of 30 feet; let the dead load be 1,200 pounds per linear foot, one-third of which is assumed to be applied at the joints of the unloaded chord. Let it be required to calculate the

dead-load stresses in all the members cut by a plane at the section pp. This plane is extended so that it will cut all the separate systems represented in Fig. 19 (b), (c), (d), and (e), and to each system the general method of sections is applied independently. When there is a large number of joints, as in this case, it is convenient to number them from each end of the truss, as shown in the lower part of Fig. 19 (a).

The dead panel load is equal to

$$\frac{1{,}200 \times 12.5}{2} = 7{,}500 \text{ pounds}$$

of which there is 5,000 at each of the joints b, c, d, etc., and 2,500 at each of the joints B, C, D, etc. The half top-panel loads at A and A' will only affect the stresses in aA, and $a'A'$; they need not be considered in this example. The figure gives

$$\csc H = \frac{\sqrt{30^2 + 25^2}}{30} = \frac{39.05}{30} = 1.30$$

In the primary system, Fig. 19 (b), the left reaction is

$$\frac{5{,}000\,(4 + 8 + 12) + 2{,}500\,(2 + 6 + 10 + 14)}{16} = 12{,}500 \text{ pounds,}$$

and the stresses in the various members are as follows:

Member	Stress, in Pounds
Ce	$(12{,}500 - 2{,}500) \times 1.30 = -13{,}000$
CG	$\dfrac{12{,}500 \times 50 - 2{,}500 \times 25}{30} = +18{,}800$
ae	$\dfrac{12{,}500 \times 25}{30} = -10{,}400$

In the secondary system, Fig. 19 (c), the left reaction is

$$\frac{5{,}000\,(2 + 6 + 10 + 14) + 2{,}500\,(4 + 8 + 12)}{16} = 13{,}750 \text{ pounds,}$$

and the stresses in the various members are as follows:

Member	Stress, in Pounds
cE	$(13{,}750 - 5{,}000) \times 1.30 = +11{,}400$
AE	$\dfrac{13{,}750 \times 25}{30} = +11{,}500$
cg	$\dfrac{13{,}750 \times 50 - 5{,}000 \times 25}{30} = -18{,}800$

In the tertiary system, Fig. 19 (*d*), the left reaction is

$$\frac{5{,}000\,(3+7+11+15)+2{,}500\,(1+5+9+13)}{16}$$
$$= 15{,}625 \text{ pounds,}$$

and the stresses in the various members are:

Member	Stress, in Pounds
bD	$(15{,}625-5{,}000)\times 1.30 = +13{,}800$
AD	$\dfrac{15{,}625\times 12.5}{30} = +\ 6{,}500$
bf	$\dfrac{15{,}625\times 37.5-5{,}000\times 25}{30} = -15{,}400$

In the quaternary system, Fig. 19 (*e*), the left reaction is

$$\frac{5{,}000\,(1+5+9+13)+2{,}500\,(3+7+11+15)}{16}$$
$$= 14{,}375 \text{ pounds,}$$

and the stresses are:

Member	Stress, in Pounds
Bd	$(14{,}375-2{,}500)\times 1.30 = -15{,}400$
BF	$\dfrac{14{,}375\times 37.5-2{,}500\times 25}{30} = +15{,}900$
ad	$\dfrac{14{,}375\times 12.5}{30} = -\ 6{,}000$

The actual stresses in the various members are as follows:

Member		Stress, in Pounds
bD		$+13{,}800$
cE		$+11{,}400$
Ce		$-13{,}000$
Bd		$-15{,}400$
CD	sum of stresses in AD, AE, BF, CG, $= 6{,}500+11{,}500+15{,}900+18{,}800 =$	$+52{,}700$
cd	sum of stresses in ad, ae, bf, cg, $= -6{,}000-10{,}400-15{,}400-18{,}800 =$	$-50{,}600$

EXAMPLES FOR PRACTICE

1. Using the same dead loads and dimensions as in Fig. 19, and a live load of 2,000 pounds per linear foot, determine the maximum and minimum stresses due to combined live and dead load in all the members cut by a plane at the section q.

Ans.

Member	Stress, in Pounds Maximum	Stress, in Pounds Minimum
HI	+ 268,800	+ 99,500
Hh'	− 11,000	+ 5,300
gI	+ 9,750	− 6,500
Gi	− 15,400	+ 800
$H'h$	− 11,000	+ 5,300
hi	− 256,300	− 97,400

2. With the same data as in example 1, find the maximum stress in aA due to combined live and dead load. Assume that the half-dead panel load at A is included in the secondary system.

Ans. + 83,750 lb.

3. With the same data as in example 1, find, by the method of joints, the dead-load stress in the member FG. Ans. + 90,100 lb.

THE PRATT TRUSS

45. Description.—The **Pratt truss**, Fig. 20 (a), is a simple type of truss in which the web members are alternately vertical and inclined; there is a vertical web member at each panel point and an inclined member in each panel, connecting the top of one vertical with the bottom of the next. The Pratt truss is used in deck, through, and half-through bridges; is built pin-connected more frequently than riveted; and is especially adapted to span lengths of 100 to 250 feet. For the longer spans, multiple systems of web or subdivided panels are sometimes built, although such forms are going out of use.

46. Diagonals.—In all except the end panels, *the diagonals are designed to resist tension only.* It was shown in the analysis of the Warren truss that positive shear causes tension in diagonals that slope upwards to the left, and negative shear tension in diagonals that slope upwards to the right. Therefore, in order to have the diagonals in tension, they must slope upwards to the left in the panels where the

shear is positive, and upwards to the right in the panels where the shear is negative. In the panels near the center, in which the maximum combined shear is opposite to the minimum shear, two diagonals will be required, one sloping in each direction. In a panel where there are two diagonals, one of them, the main diagonal, will be in tension when the combined shear is a maximum; the other, called a **counter**, will be in tension when the combined shear is a minimum. When any loading causes tension in one of these diagonals,

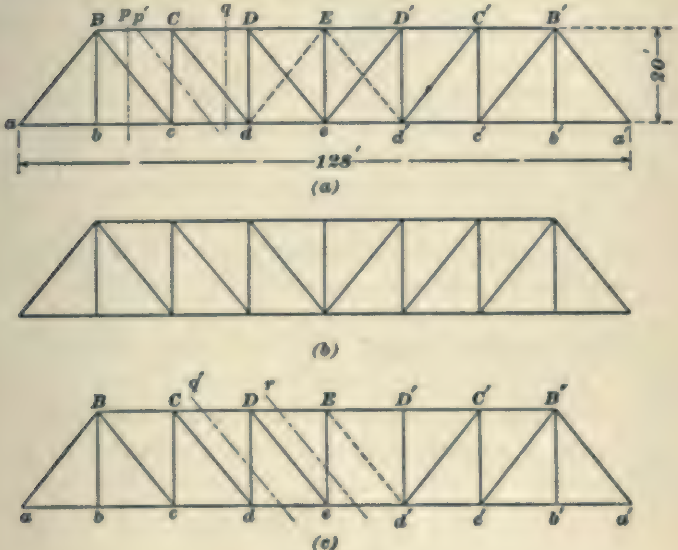


Fig. 20

that diagonal is said to be in action, and the stress in the other is assumed to be zero; the latter diagonal is said to be out of action and need not be considered as part of the truss for that loading. For example, when there is a full load on the truss shown in Fig. 20 (*a*), the shear in panel *de* is positive; there is tension in the main diagonal *De*, and the counter *dE* is out of action; the shear in panel *ed'* is negative, there is tension in the main diagonal *eD'*, and the counter *Ed'* is out of action. The members of the truss that are in action for full load are shown in Fig. 20 (*b*); as

the counters are out of action, they are omitted from the diagram.

47. Method of Calculation.—The stresses in the members of the simple Pratt truss can be found analytically or graphically. The work of calculation by either of the analytic methods is so simple that the graphic method will not be employed. The work of calculation can be best illustrated by the consideration of special cases.

THE THROUGH PRATT TRUSS WITH AN EVEN NUMBER OF PANELS

48. Description.—In Fig. 20 (*a*) is represented a through Pratt truss that has an even number of panels, the vertical member Ee being the center vertical. The span is 128 feet, and the height 20 feet. The end posts aB and $a'B'$ are compression members; the verticals Bb and $B'b'$ are tension members, and are called the **hip verticals** or **end suspenders**; all other verticals are compression members. The members dE and Ed' are the counters.

It will be assumed that the dead load is 800 pounds, all of which is applied at the joints of the loaded chord, and that the live load is 2,000 pounds per linear foot of bridge.

49. Panel Loads and Reactions.—The dead panel load is

$$\frac{800}{2} \times 16 = 6{,}400 \text{ pounds},$$

and each dead-load reaction is

$$\frac{6{,}400 \times 7}{2} = 22{,}400 \text{ pounds}$$

The live panel load is

$$\frac{2{,}000}{2} \times 16 = 16{,}000 \text{ pounds},$$

and each live-load reaction for full load is

$$\frac{16{,}000 \times 7}{2} = 56{,}000 \text{ pounds}$$

50. Chord Stresses.—To calculate the stress in any chord member, the truss may be considered cut by a surface

that intersects three members, one of which is the member whose stress is desired. The stress in the member is then equal to the bending moment on the truss at the intersection of the two other members, divided by the height of the truss. For example, for the member $B C$, Fig. 20 (a), the truss may be cut by the plane p, or by the plane p'; in either case, the center of moments is at c, and the stress in $B C$ is equal to the bending moment at c divided by the height.

For the member $c d$, the truss may be cut by either p' or q; in either case, the center of moments is at C. As C is vertically over c, the bending moments at these two points are the same; therefore, the stress in $B C$ is numerically equal to the stress in $c d$. This may also be proved by applying the equation $\Sigma X = \Sigma S \cos H = 0$ to all the forces to the left of section p': as the only horizontal forces are the stresses in $B C$ and $c d$, they must be equal and opposite. In like manner, it may be shown that the stress in $C D$ is equal to the stress in $d e$.

In calculating the moments at the various points, the work may be simplified by taking the panel length as the unit of length, that is, by expressing the lever arms in panel lengths, and multiplying the result by the panel length in feet. Thus, the moment of R_1 about d may be written $R_1 \times 3$, the distance from the line of action of R_1 to d being 3 panels. Likewise, the moments of the loads at b and c, with respect to d, are, respectively, $W \times 2$ and $W \times 1$. The resultant moment, referred to the panel length as the unit of length, is

$$R_1 \times 3 - W \times 2 - W \times 1$$

The moment (in foot-pounds or foot-tons, as the case may be) is obtained by multiplying this result by the panel length, 16; thus,

$$\text{moment at } d = (R_1 \times 3 - W \times 2 - W \times 1) \times 16$$

The dead-load chord stresses are as follows:

Stress in $a b$, $b c$ (center of moments at B),

$$\frac{(22{,}400 \times 1) \times 16}{20} = -17{,}900 \text{ pounds}$$

Stress in $c d$ (center of moments at C),

$$\frac{(22{,}400 \times 2 - 6{,}400 \times 1) \times 16}{20} = -30{,}700 \text{ pounds}$$

Stress in *de* (center of moments at *D*),

$$\frac{[22{,}400 \times 3 - 6{,}400\,(2+1)] \times 16}{20} = -38{,}400 \text{ pounds}$$

Stress in *BC* (stress in *cd*), + 30,700 pounds.

Stress in *CD* (stress in *de*), + 38,400 pounds.

Stress in *DE* (center of moments at *e*),

$$\frac{[22{,}400 \times 4 - 6{,}400\,(3+2+1)] \times 16}{20} = +41{,}000 \text{ pounds}$$

The dead-load chord stresses are shown in Fig. 21.

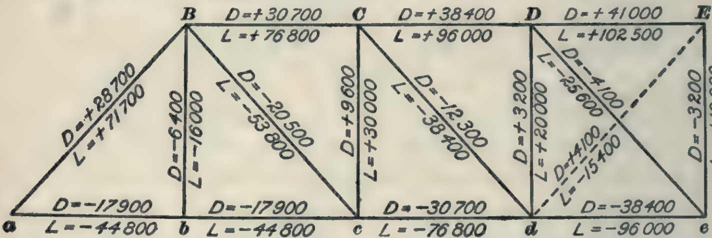


FIG. 21

The live-load chord stresses may be found as above; but, as the dead load is all applied at the joints of the loaded chord, they are more conveniently determined by multiplying the dead-load stresses by the ratio of live to dead load, which is $\frac{2{,}000}{800}$, or $\frac{5}{2}$. The results are as follows:

MEMBER	STRESS, IN POUNDS
ab, *bc*	$17{,}900 \times \frac{5}{2} = -\ 44{,}800$
cd	$30{,}700 \times \frac{5}{2} = -\ 76{,}800$
de	$38{,}400 \times \frac{5}{2} = -\ 96{,}000$
BC	$30{,}700 \times \frac{5}{2} = +\ 76{,}800$
CD	$38{,}400 \times \frac{5}{2} = +\ 96{,}000$
DE	$41{,}000 \times \frac{5}{2} = +\ 102{,}500$

The live-load chord stresses are shown in Fig. 21, and the combined chord stresses are shown in Fig. 22. The

maximum stresses are obtained by adding the dead- and live-load stresses; the minimum stresses are the dead-load stresses.

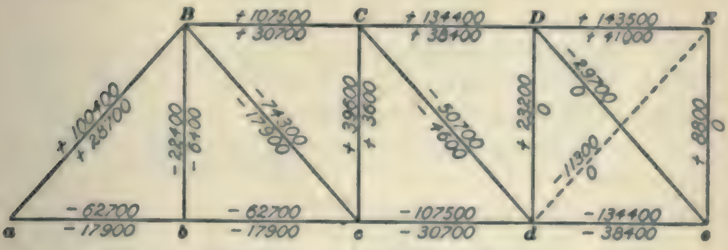


FIG. 22

51. Dead-Load Shears.—As the chords are horizontal, the vertical component of the stress in any web member is equal to the shear on the plane of section that cuts such member and two chord members. Thus, the vertical component in Cd, Fig. 20 (a), is equal to the shear on section q. Since in this case there is no load at C, the shear on p' is equal to the shear on q, and it follows that the stress in Cc is equal to the vertical component of the stress in Cd. Likewise, the stress in Dd is equal to the vertical component in De.

The dead-load shears are as follows:

Panel	Shear, in Pounds
$a\,b$	+ 22,400
$b\,c$	+ 16,000
$c\,d$	+ 9,600
$d\,e$	+ 3,200
$e\,d'$	− 3,200

and likewise for the remaining panels.

52. Live-Load Shears.—The approximate maximum positive live-load shear in any panel occurs when the truss is loaded on the *right* of the panel (see *Stresses in Bridge Trusses*, Part 1). The approximate values of the maximum positive live-load shears are as follows:

Panel $a\,b$ (loads at b, c, d, e, d', c', and b'),

$$\frac{16{,}000\,(1+2+3+4+5+6+7)}{8} = 56{,}000 \text{ pounds}$$

Panel bc (loads at c, d, e, d', c', and b'),

$$\frac{16{,}000\,(1+2+3+4+5+6)}{8} = 42{,}000 \text{ pounds}$$

Panel cd (loads at d, e, d', c', and b'),

$$\frac{16{,}000\,(1+2+3+4+5)}{8} = 30{,}000 \text{ pounds}$$

Panel de (loads at e, d', c', and b'),

$$\frac{16{,}000\,(1+2+3+4)}{8} = 20{,}000 \text{ pounds}$$

Panel ed' (loads at d', c', and b'),

$$\frac{16{,}000\,(1+2+3)}{8} = 12{,}000 \text{ pounds}$$

Panel $d'c'$ (loads at c' and b'),

$$\frac{16{,}000\,(1+2)}{8} = 6{,}000 \text{ pounds}$$

Panel $c'b'$ (load at b'),

$$\frac{16{,}000 \times 1}{8} = 2{,}000 \text{ pounds}$$

In $b'a'$, there can be no positive shear.

The maximum negative shear in any panel is numerically the same as the maximum positive shear in the corresponding panel at the other end of the truss.

53. Combined Shears.—By combining dead-load with positive and negative live-load shears for the left half of the truss, the following results are obtained:

Panel	Dead-Load Shear Pounds	Positive Live-Load Shear Pounds	Negative Live-Load Shear Pounds	Maximum Shear (Dead + Positive Live-Load) Pounds	Minimum Shear (Dead + Negative Live-Load) Pounds
ab	+ 22,400	+ 56,000		+ 78,400	+ 22,400
bc	+ 16,000	+ 42,000	− 2,000	+ 58,000	+ 14,000
cd	+ 9,600	+ 30,000	− 6,000	+ 39,600	+ 3,600
de	+ 3,200	+ 20,000	− 12,000	+ 23,200	− 8,800

54. Exact Live-Load Shears.—For purposes of comparison, the exact maximum live-load shears may be

calculated from the formula given in *Stresses in Bridge Trusses*, Part 1; namely,

$$V'' = W'' \times \frac{m^2}{2(n-1)}$$

In the present case, W'', the panel load, is equal to 16,000 pounds, and n is equal to 8. For the panel $a\,b$, $m = 7$; for the panel $b\,c$, $m = 6$; etc. The exact shears, the approximate shears, and the differences are as follows:

Panel	Exact Shear Pounds	Approximate Shear Pounds	Difference Pounds
$a\,b$	56,000	56,000	
$b\,c$	41,100	42,000	900
$c\,d$	28,600	30,000	1,400
$d\,e$	18,300	20,000	1,700
$e\,d'$	10,300	12,000	1,700
$d'\,c'$	4,600	6,000	1,400
$c'\,b'$	1,100	2,000	900
$b'\,a'$			

The approximate shears are greater than the exact in every panel except the two end panels. This is on the safe side, and, as the differences are not great, the approximate values may be used.

55. Maximum and Minimum Stresses in the Diagonals.—The maximum stresses in the diagonals are found from the maximum combined shears. The stress in any diagonal is equal to the combined shear in the panel in which the member is located multiplied by csc H, which in this case is

$$\frac{\sqrt{16^2 + 20^2}}{20} = 1.28$$

In all panels except $d\,e$, both the maximum and minimum combined shears are positive. In panel $d\,e$, they are of opposite kinds; when the truss is loaded on the right of this panel, the combined shear is positive; when loaded on

the left, it is negative. This reversal of shear requires the use of two diagonals in panel de. As explained in Art. **46**, the diagonal De, which slopes upwards toward the left, is the main diagonal and is in tension when the shear in panel de is positive; the diagonal dE, which slopes upwards toward the right, is the counter, and is in tension when the shear is negative. The maximum combined shear in the panel de is positive and equal to 23,200 pounds; then, the maximum tension in De is equal to $23{,}200 \times 1.28 = 29{,}700$ pounds. The minimum combined shear in panel de is negative and equal to 8,800 pounds; then the maximum tension in dE is equal to $8{,}800 \times 1.28 = 11{,}260$ pounds. The minimum stress in each diagonal in panel de is equal to zero, as it is assumed that when one is in action the other is out of action.

56. From the foregoing, it follows that the maximum tension in any main diagonal to the left of the center is equal to the maximum positive combined shear in the panel in which the diagonal is located, multiplied by csc H. When the minimum combined shear in any panel is positive, the minimum tension in the diagonal in that panel is equal to the minimum combined shear multiplied by csc H; when the minimum combined shear is negative, a counter is required, the maximum tension in which is equal to the minimum combined shear multiplied by csc H; in the latter case, the minimum tension in both the main diagonal and the counter are equal to zero.

57. Maximum Stresses in the Verticals.—The stress in the hip vertical Bb is found by considering the joint b; the maximum combined stress in Bb is tension, and equal to the sum of a dead and a live panel load, or 22,400 pounds. The stress in Cc is found by considering joint C. As the only vertical forces acting at this joint are the stresses in Cc and Cd, the maximum combined stress in Cc is compression and equal to the vertical component of the maximum combined stress in Cd, or 39,600 pounds. In like manner, the maximum combined stress in Dd is found to be 23,200 pounds compression, and in Ee, 8,800 pounds compression.

The stress in Ee is a maximum when joints d', c', and b' are loaded with live load. Under this condition of loading, the combined shear in panel ed' is positive, counter Ed' is in action, and main diagonal eD' is out of action. The members of the truss that are in action for this loading are shown in Fig. 20 (c). The stress in Ee is equal to the shear on section r, which is the maximum positive shear in panel ed', or 12,000 − 3,200 = + 8,800 pounds compression.

When a portion of the dead load is assumed to be applied at the joints of the unloaded chord, the stress in any vertical to the left of the center, except the hip vertical, is equal to the vertical component of the stress in the diagonal in the panel to the right, plus the load at the joint of the unloaded chord.

58. Minimum Stresses in the Verticals.—The minimum stress in Bb is tension, and equal to a dead panel load, or 6,400 pounds. The combined stress in Cc is compression, and equal to the vertical component of the minimum combined stress in Cd, or 3,600 pounds. The minimum stress in Dd is zero, and occurs when the stress in the diagonal De is a minimum, that is, when the counter dE is in action and the main diagonal De is out of action. In like manner, the minimum stress in Ee is zero, and occurs when the main diagonals De and eD' are both in action, the stresses in the counters dE and $d'E$ being zero.

59. Combined Stresses.—The maximum and minimum stresses are shown in Fig. 22; the dead and live-load stresses, in Fig. 21. In the counter dE, Fig. 21, there is shown a compressive stress of 4,100 pounds due to dead load; and in Ee, a tensile stress of 3,200 pounds. These are the stresses that would occur in dE and Ee if the main diagonal De in the panel de were omitted when there is no live load on the truss. No compression can actually occur in dE, and no tension in Ee, the values given being the amounts by which the live-load stresses in those members are reduced by the dead load when the main diagonal De is out of action.

60. Odd Number of Panels.—The Pratt truss represented in Fig. 23 has an odd number of panels. In this truss, there is a center panel in which the dead-load shear is zero.

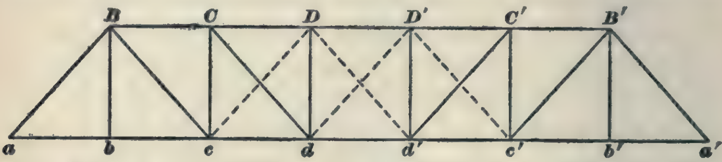


FIG. 23

The dead-load stresses in the two diagonals in this panel are, therefore, equal to zero, and both may be considered as counters. Otherwise, this truss is the same as the one already analyzed.

61. Chord Stresses for Partial Load.—It is sometimes necessary to compute the stress in a chord member due to a live load over a portion of the span. In the panels where there are no counters, the center of moments for any chord member will be the same for a partial as for a full load. In the panels where there are counters, the center of moments for the chord members depends on whether the counter or the main diagonal is in action for the specified loading. If the main diagonal is in action, the center of moments will be at the intersection of the main diagonal with the opposite chord; if the counter is in action, the center of moments will be at the intersection of the counter with the opposite chord.

EXAMPLE.—Let it be required to calculate the stress in the chord member $e\,d'$, if the through Pratt truss shown in Fig. 20 (a) carries a dead load of 6,400 pounds at each panel point, and a live load of 16,000 pounds at each of the panel points e, d', c', and b'.

SOLUTION.—The dead-load reaction at the left end is

$$\frac{7 \times 6{,}400}{2} = 22{,}400 \text{ lb.}$$

The dead-load shear in the panel $e\,d'$ is

$$22{,}400 - 4 \times 6{,}400 = -3{,}200 \text{ lb.}$$

The live-load reaction at the left end is

$$\frac{16{,}000 \times (1 + 2 + 3 + 4)}{8} = 20{,}000 \text{ lb.}$$

The live-load shear in the panel $e\,d'$ is

$$20{,}000 - 16{,}000 = +4{,}000 \text{ lb.}$$

The combined shear in the panel $e\,d'$ is

$$-3{,}200 + 4{,}000 = +800 \text{ lb.}$$

The combined shear is positive, and causes tension in the member sloping upwards to the left, which is the counter $d'E$. Therefore, the counter $d'E$ is in action for the specified loading, as shown in Fig. 20 (c), and the center of moments for $e\,d'$ is at E, the intersection of $d'E$ and $E\,D'$. Then, the stress in $e\,d'$ is equal to the moment at E divided by the height of the truss; or, the tension in $e\,d'$ is

$$\frac{[22{,}400 \times 4 - 6{,}400\,(1 + 2 + 3) + 20{,}000 \times 4] \times 16}{20} = 104{,}960 \text{ lb.} \quad \textbf{Ans.}$$

THE DECK PRATT TRUSS

62. Description.—The deck Pratt truss may be supported in any of the ways shown in Figs. 24, 25, and 26.

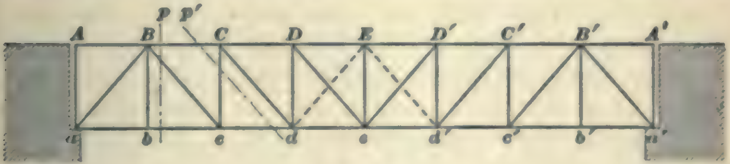


FIG. 24

These three trusses are alike except for the end panels. In Fig. 24, the truss is supported at the point a in the same way

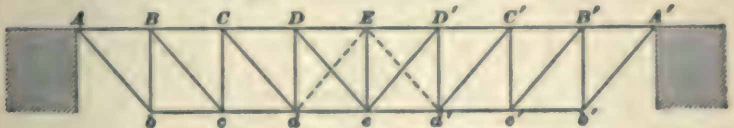


FIG. 25

as the through Pratt truss; the loads in the end panel are supported by stringers $A\,B$, the ends of which either rest on

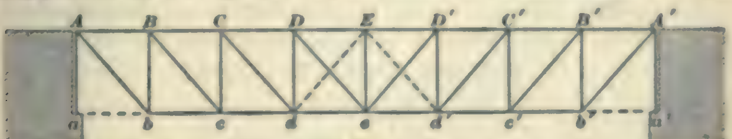


FIG. 26

the masonry at A or are supported by vertical members $a\,A$ In this truss the inclined member $a\,B$ is the end post, and is

a compression member. In Fig. 25, the truss is supported at the point A, and the end diagonal $A\,b$ is a tension member. In Fig. 26, the truss is supported at the point a; the vertical $a\,A$ is in compression, and is called the *vertical end post;* the web diagonal $A\,b$ is in tension; the stress in $a\,b$ due to dead and live loads is zero, this member being inserted for the sake of stiffness.

63. Chord Stresses.—Using the same loads and dimensions as for the through Pratt truss in Fig. 20 (a), the stress in any chord member of the deck Pratt truss will be the same as the corresponding member of the through Pratt truss. For example, the stress in $C\,D$, Fig. 20 (a), is equal to the moment at d divided by the height of the truss; also, the stress in $C\,D$, Figs. 24, 25, and 26, is equal to the moment at d divided by the height of the truss. In Figs. 25 and 26, the stress in $A\,B$ equals the stress in $b\,c$. For any other dimensions and loading, the chord stresses may be found in exactly the same way as for the through Pratt truss. The maximum chord stresses occur when there is a full live load; the minimum, when there is no live load.

64. Stresses in Diagonal Members.—The end diagonal in Fig. 24 is a compression member; the end diagonals in Figs. 25 and 26 are tension members. The vertical component of the maximum stress in the end diagonal is equal to the maximum positive shear in the end panel; the vertical component of the minimum stress is equal to the dead-load shear. The stresses in all the other diagonals may be found in exactly the same way as for the through truss.

65. Stresses in Vertical Members.—The stresses in the verticals are different from those in the corresponding members of the through truss. The vertical $B\,b$, Fig. 24, is the hip vertical; the stress in $B\,b$ is equal to zero, or to the dead load at b, if any. The maximum stress in $A\,a$, Fig. 26, is equal to the left reaction when there is a full live load; the minimum stress in $A\,a$ is equal to the left reaction when there is no live load. In calculating the

stress in *Aa*, one-half of a panel load must be applied at *A*, as this is carried to the abutment by *Aa*. The stress in any other vertical is, in general, equal to the shear on a plane of section cutting that vertical and two chord members. Thus, the stress in *Cc*, Fig. 24, is equal to the shear in the plane of section *p'*. Then, the maximum compression in any vertical on the left of the center (except the hip vertical and vertical end post) is equal to the maximum positive shear in a plane cutting that vertical and the two chord members between which the vertical lies. This occurs when the joint at the top of the member and all joints to the right are loaded.

The maximum positive combined shear in the panel to the left of the center vertical may be less than the sum of a dead and a live panel load, in which case the maximum compression in the vertical will be equal to the sum of a dead and a live panel load. In this case, the shear in the panel to the right of the vertical will be negative, and the main diagonals in both center panels will be in action; as the stresses in the two counters that meet at the top of the center vertical will then be equal to zero, the vertical will simply support the load at its upper joint.

When the minimum combined shear in any panel on the left of the center is positive and *greater* than a dead panel load, the minimum stress in the vertical on the right of such panel is equal to the minimum combined shear on the plane cutting such vertical and the two chord members between which it lies. When the minimum combined shear is positive and *less* than a dead panel load, the shear in the panel to the right of the vertical is negative. The diagonal sloping upwards to the left will be in action in the panel on the left; the diagonal sloping upwards to the right, in the panel on the right, and the stress in the diagonal or diagonals that meet the vertical at the top will be zero. Then, the only vertical forces acting at the top of the vertical member will be the dead panel load and the stress in the member; therefore, the minimum stress in such a vertical member will be equal to a dead panel load.

When the minimum shear in the panel to the left of any vertical is negative, the shear in that panel may have any value between the minimum shear, which is negative, and the maximum shear, which is positive. Therefore, under some conditions of loading, the shear in the panel to the left will be positive and less than a dead panel load, and then the stress in the vertical will, as shown, be a minimum and equal to a dead panel load. In like manner, it may be shown that the minimum stress in any vertical between this member and the center will be equal to a dead panel load.

EXAMPLE.—Let it be required to calculate the maximum and minimum stresses in the verticals of the deck Pratt truss shown in Fig. 25, using the same dimensions and loads as for the through truss shown in Fig. 20 (*a*) and described in Art. **48**.

SOLUTION.—The maximum and minimum shears are the same as for the through truss, and are given in Art. **53**. They are as follows:

Panel	Maximum Shear Pounds	Minimum Shear Pounds
AB	+ 78,400	+ 22,400
BC	+ 58,000	+ 14,000
CD	+ 39,600	+ 3,600
DE	+ 23,200	− 8,800

The maximum stresses (which are the sum of the dead- and live-load stresses) are as follows:

MEMBER	STRESS, IN POUNDS
Bb	+ 22,400 + 56,000 = + 78,400
Cc	+ 16,000 + 42,000 = + 58,000
Dd	+ 9,600 + 30,000 = + 39,600
Ee	+ 3,200 + 20,000 = + 23,200

The maximum stress in Ee is equal to the maximum positive shear in panel DE, as this is greater than the sum of a full dead and a full live panel load (22,400 lb.).

The minimum stresses are as follows:

MEMBER	STRESS, IN POUNDS
Bb	+ 22,400
Cc	+ 14,000
Dd	+ 6,400
Ee	+ 6,400

The minimum shear in panel CD is positive and equal to 3,600 pounds; it occurs when joints B and C are loaded. Under this condition of loading, the load at D is a dead panel load, or 6,400 pounds. Then, the shear in panel DE for the same loading is 3,600 − 6,400 = −2,800 pounds; as this is negative, the counter dE is in action and the stress in De is zero. Therefore, the stress in Dd must be equal to the dead panel load at D, which is equal to 6,400 pounds. In like manner, it may be shown that the minimum stress in Ee is 6,400 pounds.

EXAMPLES FOR PRACTICE

1. Let Fig. 27 be a ten-panel through Pratt truss having a span length of 180 feet and a height of 25 feet. If the dead load is 1,200 pounds, one-third of which is applied at the joints of the unloaded chord, and the live load is 2,400 pounds, per linear foot of bridge,

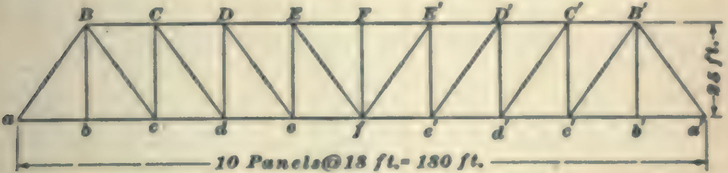


FIG. 27

what are: (*a*) the maximum stresses in the counters and main diagonals in all the panels in which counters are required? (*b*) the maximum and minimum stresses due to combined live and dead load in the diagonals aB and Bc? (*c*) the maximum and minimum stresses due to combined dead and live load in the verticals Bb, Dd, and Ff? (*d*) the maximum and minimum stresses due to combined dead and live load in the chord members bc, BC, and DE?

		Member	Stress, in Pounds Maximum	Stress, in Pounds Minimum
Ans.	(*a*)	Ef, fE'	− 46,600	0
		eF, Fe'	− 20,000	0
	(*b*)	aB	+ 179,600	+ 59,900
		Bc	− 142,400	− 43,900
	(*c*)	Bb	− 28,800	− 7,200
		Dd	+ 65,160	+ 6,840
		Ff	+ 19,800	+ 3,600
	(*d*)	bc	− 105,000	− 35,000
		BC	+ 186,600	+ 62,200
		DE	+ 279,900	+ 93,300

2. Let Fig. 28 be a nine-panel deck Pratt truss having a span length of 180 feet and a height of 26 feet. If the dead load is 1,200 pounds, one-third of which is applied at the joints of the unloaded chord, and the live load is 2,200 pounds, per linear foot of bridge: (*a*) what are

the maximum combined stresses in the counters and main diagonals in all the panels in which counters are required? (*b*) what are the

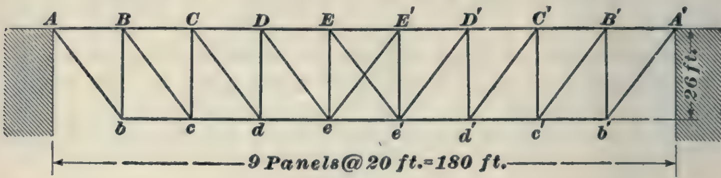


FIG. 28

maximum and minimum combined stresses in the diagonals Ab and Cd? (*c*) in the verticals Bb, Dd, and Ee? (*d*) in the chord members bc, CD, and EE'?

Ans.

	Member	Stress, in Pounds Maximum	Stress, in Pounds Minimum
(*a*)	Ee', eE'	− 30,800	0
	De, $e'D'$	− 61,400	0
	dE, $d'E'$	− 3,400	0
(*b*)	Ab	− 171,600	− 60,600
	Cd	− 95,100	− 21,000
(*c*)	Bb	+ 132,000	+ 44,000
	Dd	+ 71,300	+ 12,700
	Ee	+ 44,700	+ 8,000
(*d*)	bc	− 104,600	− 36,900
	CD	+ 235,400	+ 83,100
	EE'	+ 261,500	+ 92,300

THE HOWE TRUSS

66. Description.—The **Howe truss,** shown in Fig. 29, was one of the earliest forms of simple bridge trusses in America. As originally constructed, it had short panels with

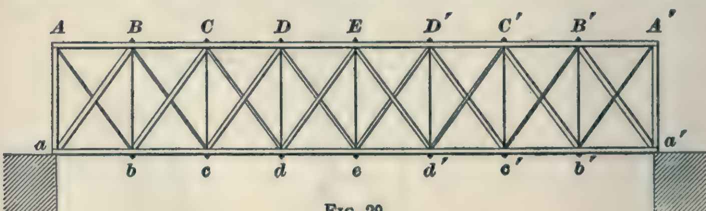


FIG. 29

two diagonals in each panel, and vertical end posts; all parts of the truss were constructed of wood, except the intermediate verticals, which were iron rods. This type of truss is still used to some extent in localities where timber is

plentiful. At present, however, all the members that receive no stress are omitted, and the lower chord is frequently constructed of steel. The diagonals are designed to resist compression only; therefore, they must slope downwards to the left in the panels in which the shear is positive; and downwards to the right in the panels in which the shear is negative. Two diagonals, one sloping in each direction, are required in each panel in which the sign of the maximum combined shear is opposite to that of the minimum combined shear. In the original trusses of this type, two diagonals were put in each panel, but the extra diagonals or counters in the panels near the ends were unnecessary. The verticals, except the vertical end post, were designed to resist tension only.

The modern forms of the Howe truss are shown in Fig. 30 as a through truss, and in Fig. 31 as a deck truss. The end posts, upper chord, and intermediate diagonals are constructed of wood; the verticals and bottom chord are of steel. The counters are shown in dotted lines, and are put in only where necessary.

67. Calculation of Stresses.—The stresses in the members are calculated in exactly the same way as the stresses in the members of the Pratt truss. The maximum stress in the center vertical Dd of the through truss shown in Fig. 30 is equal to the maximum positive shear in panel cd

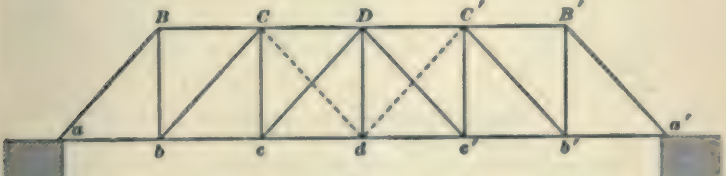


FIG. 30

(when there is no dead load at D), or to a full dead and live panel load, whichever is the greater; the minimum stress is equal to a dead panel load. The maximum stress in the center vertical dD of the deck truss shown in Fig. 31 is equal to the maximum positive shear in the panel dc'; the

minimum stress is equal to zero, and occurs when the main diagonals Cd and dC' are in action. (In reality, the stress

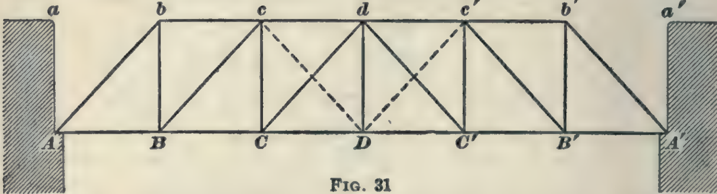


FIG. 31

in dD, under this condition of loading, is equal to the dead load at D; but, if this is all assumed at the joints of the loaded chord, the stress in dD will be zero.)

EXAMPLE FOR PRACTICE

Let Fig. 30 be a six-panel through Howe truss having a span length of 90 feet, consisting of six 15-foot panels, and a height of 12 feet. If the dead load is 800 pounds, all applied at the loaded chord, and the live load is 2,000 pounds, per linear foot of bridge; find. (*a*) the maximum and minimum combined stresses in the chord members BC and bc; (*b*) the maximum combined stresses in both the main diagonal and the counter in each panel to the left of the center in which a counter is required; (*c*) the maximum and minimum combined stresses in the verticals Bb and Dd.

Ans.

	Member	Stress, in Pounds Maximum	Stress, in Pounds Minimum
(*a*)	BC	+ 65,600	+ 18,800
	bc	− 105,000	− 30,000
(*b*)	cD	+ 28,800	0
	Cd	+ 7,200	0
(*c*)	Bb	− 52,500	− 15,000
	Dd	− 21,000	− 6,000

THE WHIPPLE TRUSS

68. Description.—If the simple type of Pratt truss is used for the longer spans to which the Pratt truss is adapted, it will be impossible to make use of an economical height of truss and inclination of diagonal without using very long panels, thereby making the floor system very heavy. For such spans, it is convenient to use a modified form of the Pratt truss, making use of multiple systems of web or

subdivided panels. The **Whipple truss,** sometimes called the **double-intersection Pratt truss,** is a modified form of the Pratt truss, in which there are two systems of web members, the diagonals running across two panels, as shown in Fig. 32 (*a*). The two systems of web members are shown in full and dotted lines. There is a large number of these trusses in use at the present time, but they are now

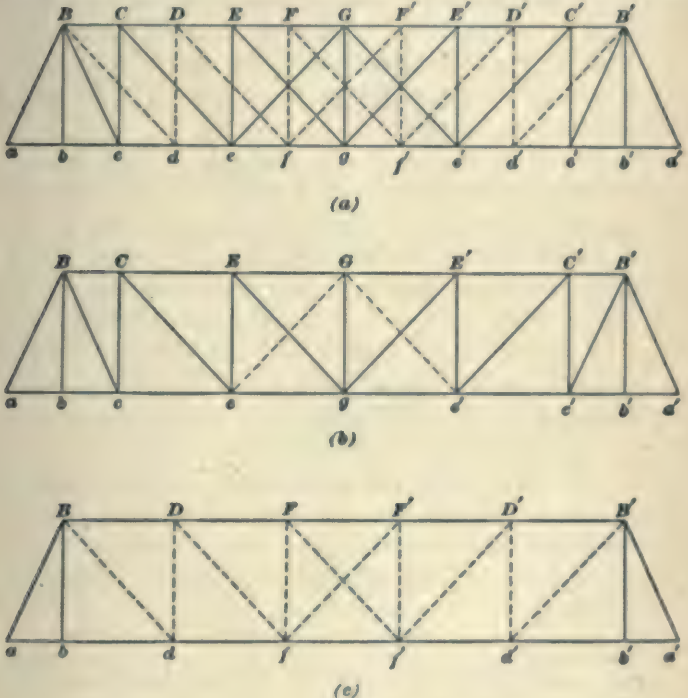


FIG. 32

avoided by the best engineers, because the actual stresses in some members cannot be determined accurately.

69. Method of Calculation.—Applying the principles of the method of sections to the truss shown in Fig. 32 (*a*), it will be seen that, if the truss is cut by a plane in any panel except the end panel, the plane will cut at least four members in which the stresses are unknown, and it will be

impossible to determine those stresses by the principles of statics, unless some assumption is made regarding the distribution of the stresses among the members. It is customary to assume that the truss is composed of two trusses lying in the same plane and having the chords and end posts in common. The stresses in each truss are supposed to be caused by the loads that are directly applied to it. The two systems in this case are shown separated in Fig. 32 (*b*) and (*c*). For convenience of reference, the system shown in full lines in Fig. 32 (*b*) will be called the *primary system*, and the system shown in dotted lines in Fig. 32 (*c*) will be called the *secondary system*.

The joints B, b, B', and b' are common to both systems and one-half of each of the loads applied at those points may be assumed to be supported by each system. The stresses may be found in all the members of each system, for the loads that come on that system, in exactly the same way in which the stresses are found in the single-intersection Pratt truss. As the chords, end posts, and hip verticals are common to both systems, the stresses found in those members in each system are partial stresses, or component stresses, and must be added together to get the actual stresses. As each of the web members, except the end posts and hip verticals, occurs in but one system, the stresses found in them are the actual stresses. The assumption that the two web systems act independently is not strictly correct, but the resulting error is probably very small.

EXAMPLE.—Let it be required to calculate the maximum stresses in the chord members CD, DE, ef, and fg, and in the web members Bc, Ee, eG, Dd, and Df, in the twelve-panel through Whipple truss shown in Fig. 32 (*a*), having a span length of 216 feet, and a height of 36 feet, when the dead load is 1,000 pounds (all applied at the joints of the loaded chord), and the live load is 1,600 pounds, per linear foot of bridge. The dead and the live panel load are 9,000 and 14,400 pounds, respectively; at the joints b and b', the panel loads for each system are to be taken at one-half these values.

SOLUTION.—*Chord Stresses*.—The truss may be separated into the primary system shown in Fig. 32 (*b*) and the secondary system shown in Fig. 32 (*c*). To find the stresses in CD, DE, ef, and fg, it is

necessary to calculate the stresses in CE and eg of the primary system, and in BD, DF, df, and ff' of the secondary system. The total dead load on the primary system is $9{,}000 \times 6$, and each reaction is equal to

$$\frac{9{,}000 \times 6}{2} = 27{,}000 \text{ lb.}$$

The dead-load chord stresses are as follows:

$$\text{Stress in } CE = \frac{(27{,}000 \times 4 - 4{,}500 \times 3 - 9{,}000 \times 2) \times 18}{36} = +38{,}250 \text{ lb.}$$

The stress in eg is equal to the stress in $CE = -38{,}250$ lb.

The total dead load on the secondary system is $9{,}000 \times 5$, and each reaction is equal to

$$\frac{9{,}000 \times 5}{2} = 22{,}500 \text{ lb.}$$

The dead-load chord stresses are as follows:

Member		Stress, in Pounds
BD	$\frac{(22{,}500 \times 3 - 4{,}500 \times 2) \times 18}{36}$	$= +29{,}250$
DF	$\frac{(22{,}500 \times 5 - 4{,}500 \times 4 - 9{,}000 \times 2) \times 18}{36}$	$= +38{,}250$
df	(stress in BD)	$-29{,}250$
ff'	(stress in DF)	$-38{,}250$

Then, the dead-load chord stresses in this truss are:

Member	Stress, in Pounds
CD	$38{,}250 + 29{,}250 = +67{,}500$
DE	$38{,}250 + 38{,}250 = +76{,}500$
ef	$38{,}250 + 29{,}250 = -67{,}500$
fg	$38{,}250 + 38{,}250 = -76{,}500$

As all the dead load is applied at the joints of the loaded chord, the live-load stresses will be equal to the dead-load stresses multiplied by $\frac{1{,}600}{1{,}000}$, and the maximum combined stresses will be equal to the dead load stresses multiplied by $\frac{2{,}600}{1{,}000}$, or by 2.6.

The maximum combined stresses are as follows:

Member	Maximum Stress, in Pounds
CD	$+67{,}500 \times 2.6 = +175{,}500$
DE	$+76{,}500 \times 2.6 = +198{,}900$
ef	$-67{,}500 \times 2.6 = -175{,}500$
fg	$-76{,}500 \times 2.6 = -198{,}900$

Web Stresses.—The web members Bc, Ee, and eG occur in the primary system. The dead-load shears are as follows:

shear in panel $bc = 27{,}000 - 4{,}500 = 22{,}500$ lb.

shear in panel $eg = 27{,}000 - 4{,}500 - 9{,}000 \times 2 = 4{,}500$ lb.

For the member $B\,c$,

$$\csc H = \frac{\sqrt{18^2 + 36^2}}{36} = 1.118;$$

for the member $e\,G$,

$$\csc H = \frac{\sqrt{36^2 + 36^2}}{36} = 1.414.$$

The dead-load stress in $B\,c$ is $22{,}500 \times 1.118 = -25{,}200$ lb.

The dead-load stress in $E\,e$ is $+4{,}500$ lb.

The dead-load stress in $e\,G$ (counter, assuming that the main diagonal $E\,g$ is left out) is $4{,}500 \times 1.414 = +6{,}400$ lb.

The members $D\,d$ and $D\,f$ occur in the secondary system. The shear in the panel $d\,f$ is $22{,}500 - 4{,}500 - 9{,}000 = 9{,}000$ lb.

The dead-load stress in $D\,d$ is $+9{,}000$ lb.

The dead-load stress in $D\,f$ is $9{,}000 \times 1.414 = -12{,}700$ lb.

The maximum live-load stresses for the primary system are as follows, the load at b' being a half-panel load: For the member $B\,c$, with the truss loaded to c from the right end, the shear in panel $b\,c$ is

$$\frac{14{,}400\left(\frac{1}{2} + 2 + 4 + 6 + 8 + 10\right)}{12} = 36{,}600 \text{ lb.}$$

The live-load stress in $B\,c$ is $36{,}600 \times 1.118 = -40{,}900$ lb.

For the member $E\,e$, with the truss loaded to g from the right end, the shear in panel $e\,g$ is

$$\frac{14{,}400\left(\frac{1}{2} + 2 + 4 + 6\right)}{12} = 15{,}000 \text{ lb.}$$

The live-load stress in $E\,e$ is $+15{,}000$ lb.

For the counter $e\,G$, the member $e'\,G$ may be considered instead. With the truss loaded to e' from the right end, the shear in panel $g\,e'$ is

$$\frac{14{,}400\left(\frac{1}{2} + 2 + 4\right)}{12} = 7{,}800 \text{ lb.}$$

The live-load stress in $e'\,G$ is $7{,}800 \times 1.414 = -11{,}000$ lb.

The maximum live-load stresses for the secondary system are as follows, the load at b' being a half-panel load: For the members $D\,d$ and $D\,f$, with the truss loaded to f, from the right end, the shear in panel $d\,f$ is

$$\frac{14{,}400\left(\frac{1}{2} + 3 + 5 + 7\right)}{12} = 18{,}600 \text{ lb.}$$

The live-load stress in $D\,d$ is $+18{,}600$ lb.

The live-load stress in $D\,f$ is $18{,}600 \times 1.414 = -26{,}300$ lb.

The final maximum combined stresses are as follows:

Member	Stress, in Pounds
$B\,c$	$-25{,}200 - 40{,}900 = -66{,}100$
$E\,e$	$4{,}500 + 15{,}000 = +19{,}500$
$e\,G$	$-11{,}000 + 6{,}400 = -4{,}600$
$D\,d$	$9{,}000 + 18{,}600 = +27{,}600$
$D\,f$	$-12{,}700 - 26{,}300 = -39{,}000$

EXAMPLE FOR PRACTICE

Let Fig. 32 (*a*) be a twelve-panel through Whipple truss having a span length of 180 feet and a height of 30 feet. If the dead load is 1,200 pounds, all applied at the joints of the loaded chord, and the live load is 2,200 pounds, per linear foot of bridge, find: (*a*) the maximum and minimum combined stresses in the chord members cd, de, EF, and FG; (*b*) the maximum and minimum combined stresses in the web members Bd, Eg, Ff', Cc, and Ff.

Ans.

	Member	Stress, in Pounds	
		Maximum	Minimum
(*a*)	cd	− 102,000	− 36,000
	de	− 153,000	− 54,000
	EF	+ 229,500	+ 81,000
	FG	+ 229,500	+ 81,000
(*b*)	Bd	− 73,100	− 24,500
	Eg	− 30,700	0
	Ff'	− 16,500	0
	Cc	+ 41,700	+ 10,100
	Ff	+ 11,700	0

THE POST TRUSS

70. Description.—The **Post truss**, Fig. 33 (*a*), may be looked on as a modified Whipple truss with an odd number of panels in the bottom chord. This truss is seldom built at present on account of the uncertainty in the stresses. The compression web members are inclined so that their upper joints are one-half a panel nearer the center of the truss than their lower joints. The two center members, one on each side of the center panel of the lower chord, meet at the center of the upper chord. The main diagonals are tension members and slope across one and one-half panels. The diagonals shown in dotted lines are counters, and were formerly inserted in each panel, although those in the panels near the ends were not needed. At present, counters are used only where they are actually needed. There are in reality two systems of web members, but as they are connected at the center joint of the upper chord, and as the counters connect the compression web members of the two systems, it is impossible to calculate the stresses from the equations of equilibrium without making some

assumption as to their distribution. The common assumptions for this truss give only roughly approximate results. It is customary to divide the truss into two systems, as shown in full lines in Fig. 33 (*b*) and (*c*), and treat each system as an independent truss in much the same way as in the Whipple truss. It will be seen that each system is unsymmetrical, but that the two are alike end for end.

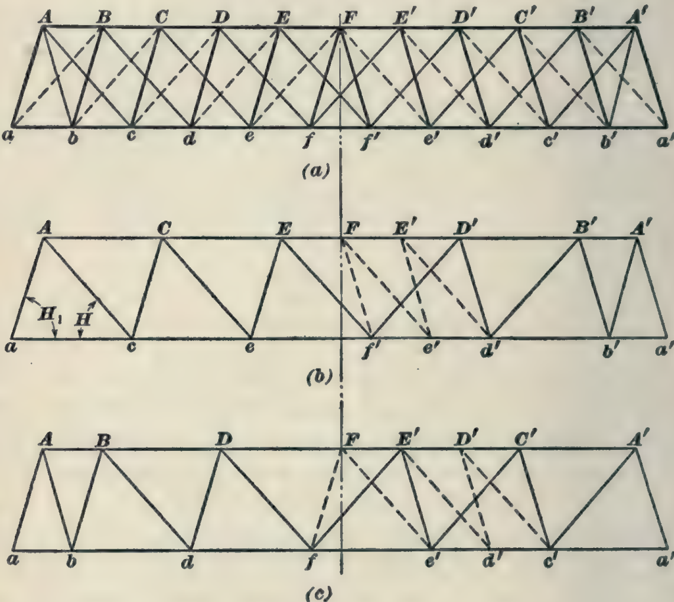


FIG. 33

71. Chord Stresses. It will be necessary to calculate the stresses in all the chord members of only one system. For example, the maximum stress in AB, Fig. 33 (*a*), is equal to the sum of the stresses in AC, Fig. 33 (*b*), and AB, Fig. 33 (*c*). But, since, when the whole truss is fully loaded, the stress in $A'B'$ is equal to that in AB, Fig. 33 (*c*), it is not necessary to calculate the latter stress separately. The stress in AC is equal to the moment at c divided by the height; the stress in $A'B'$ is equal to the moment at b' divided by the height. In like manner, other chord stresses may be found.

72. Stresses in Main Web Members.—As the systems are alike end for end, it is only necessary to analyze each system for loads on the right of the center. The member $A\,a$, Fig. 33 (a), occurs in both systems; the stress in it is equal to the sum of the reactions at a in the two systems, Figs. 33 (b) and (c), multiplied by csc H_1; or, since the reaction at a in (c) is equal to the reaction at a' in (b), the stress in $a\,A$ is equal to the sum of the two reactions of either system multiplied by csc H_1. The same is true of the stress in $A'\,a'$. The maximum stresses in the other web members may be found from the maximum shears in the respective panels. For example, the vertical component of the maximum stresses in $A\,c$ and in $A'\,c'$ is equal to the maximum positive shear in panel $a\,c$, Fig. 33 (b); in $C\,c$ and $C\,e$, $C'\,c'$ and $C'\,e'$, to the maximum positive shear in panel $c\,e$, Fig. 33 (b); in $E\,e$ and $E\,f'$, $E'\,e'$ and $E'\,f$, to the maximum positive shear in panel $e\,f'$, Fig. 33 (b).

Also, the vertical component of the maximum stresses in $A\,b$ and $A'\,b'$ is equal to the maximum positive shear in panel $a\,b$, Fig. 33 (c); in $B\,b$ and $B\,d$, $B'\,b'$ and $B'\,d'$, to the maximum positive shear in panel $b\,d$, Fig. 33 (c); in $D\,d$ and $D\,f$, $D'\,d'$ and $D'\,f'$, to the maximum positive shear in panel $d\,f$, Fig. 33 (c). The maximum positive shear in any panel obtains when all joints to the right are loaded with a live load.

73. Stresses in Counters.—The compression members $F\,f$ and $F\,f'$ may be considered as counters, as they are not in action for full load. In Fig. 33 (b), when the combined shear in panel $f'\,d'$ is positive, the members $D'\,d'$ and $D'\,f'$ may be considered out of action, and the members $d'\,E'$, $E'\,e'$, $e'\,F$, and $F\,f'$ in action. In Fig. 33 (c), when the combined shear in panel $f\,e'$ is positive, the members $e'\,E'$ and $E'\,f$ may be considered out of action, and the members $e'\,F$ and $F\,f$ in action; when the combined shear in panel $e'\,c'$ is positive, $c'\,C'$ and $C'\,e'$ may be considered out of action, and $c'\,D'$, $D'\,d'$, $d'\,E'$, and $E'\,e'$ in action. When the live load extends to e' from the right, the joints d' and b',

Fig. 33 (*b*), and e' and c', Fig. 33 (*c*), will be loaded. If the combined shear in panel fe' is positive, the vertical component of the stresses in Ff and Fe' is equal to the shear in the panel fe'; if the combined shear in the panel $f'd'$ is also positive, the vertical component of the stress in Ff' and $E'd'$ is equal to the shear in panel $f'd'$, and the vertical component of the stress in Fe' is equal to the sum of the shears in panels $f'd'$ and fe'. The stresses in the other counters may be found in like manner.

It must be understood that for this truss the foregoing method of calculation is *roughly approximate* and is given here simply to afford a means of finding stresses in trusses of this nature already built rather than as a guide in designing. The Whipple truss answers the same purpose as the Post truss, and is preferable because the approximate method of calculation gives closer results for the former than for the latter.

THE BALTIMORE TRUSS

74. Description.—The **Baltimore truss,** shown as a through bridge in Fig. 34, is in reality a Pratt truss with long panels, which are subdivided by the addition of short verticals and diagonals, such as Dd and Dc, intersecting the main diagonals half way between the top and bottom chords. This form of truss allows the use of an economical height and inclination of diagonal, without excessively long panels, and is used to a great extent at the present time for railroad and highway bridges. The short verticals (Dd, Ff, etc.), called the **subverticals,** are tension members; the short diagonals (cD, eF, etc.), called the **substruts,** are compression members; eF also acts as a tension member as the lower half of the counter eFG when that counter is in action. All the other members are similar to the corresponding members of the simple Pratt truss.

75. Chord Stresses.—The stress in any top-chord member may be found by cutting the truss by a plane that intersects the member whose stress is desired and the lower half of a main diagonal. For example, for the stress in CE,

Fig. 34 (a), the truss may be considered as cut by a plane at the section q; then the stress in CE is equal to the moment of all the forces to the left of section q about the point e

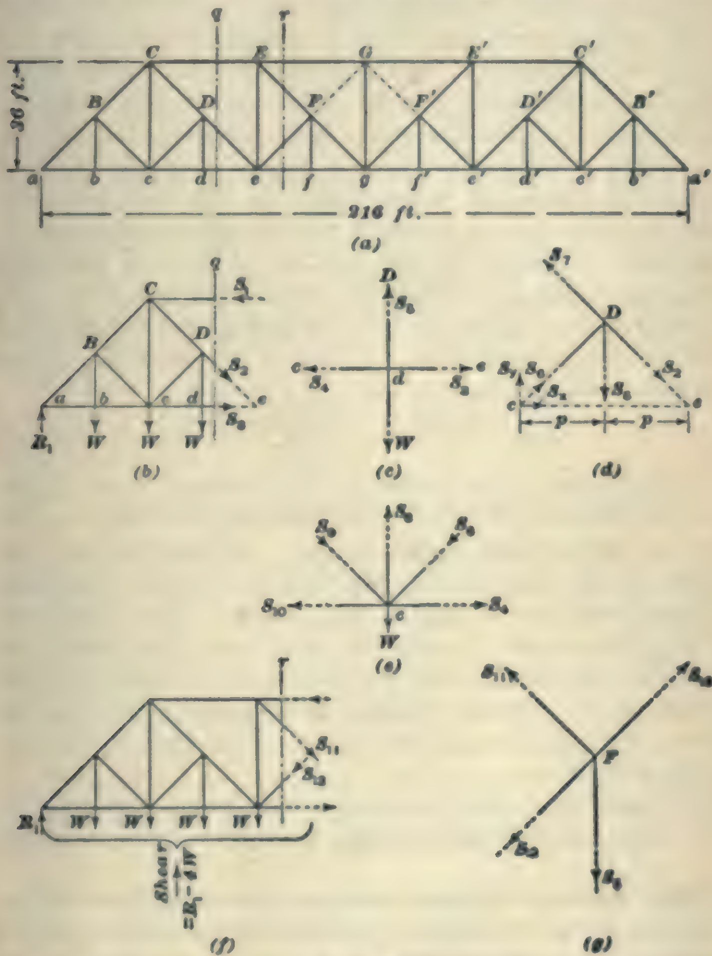


FIG. 34

divided by the height of the truss. In the present case, Fig. 34 (b),

$$S_1 = \frac{R_1 \times 4p - W(1+2+3)p}{h},$$

denoting the panel length by p, and the height of the truss by h.

The stresses in the bottom-chord members $a\,b$, $d\,e$, $f\,g$, etc. may be found by cutting the truss by a plane that intersects the member whose stress is desired and the lower half of a main diagonal. For example, for the stress in $d\,e$, Fig. 34 (a), the truss may be considered as cut by a plane at the section q; then the stress in $d\,e$ is equal to the moment of all the forces to the left of section q about the point C, divided by the height of the truss. In this case, loads at d and D, if any, have positive or right-handed moments about the point C, similar to the moment of R_1, and this must be remembered in writing the equation of moments. *The moment of the forces on the left of section q about the point C is not the bending moment on the truss at the point C.* In the present case, Fig. 34 (b),

$$S_3 = \frac{R_1 \times 2p - W \times p + W \times p}{h}$$

The stresses in the bottom-chord members $b\,c$, $c\,d$, $e\,f$, etc. may be most easily found by considering the forces acting at the intermediate panel points. For example, for the stress in $c\,d$, Fig. 34 (a), the joint d may be considered a free body, as shown in Fig. 34 (c). The only horizontal forces are S_4 and S_3; they must be equal and opposite; in other words, the stress in $c\,d$ is equal to the stress in $d\,e$. In like manner, it may be shown that the stress in $b\,c$ is equal to the stress in $a\,b$, that in $e\,f$ equal to that in $f\,g$, etc.

The maximum chord stresses obtain when there is a full live load; the minimum stresses, when there is no live load.

76. Stresses in the Subverticals.—The stress in any subvertical may be found by considering the forces acting at an intermediate panel point. For example, for the stress in $D\,d$, the joint d, Fig. 34 (c), may be considered. The only vertical forces are the stress in $D\,d$ and the panel load W; they must be equal and opposite. Therefore, *the stress in any subvertical is tension: the maximum stress is equal to the sum*

of a dead and a live panel load; the minimum stress is equal to a dead panel load.

77. Stresses in the Short Diagonals or Substruts. The stresses in the short diagonals Bc, Dc, Fe, etc. may be found by considering the forces acting at the intermediate joints B, D, F, etc. For example, for the stress in cD, Fig. 34 (a), the joint D may be treated as a free body, as shown in Fig. 34 (d). There are four forces (S, S, S, and S) acting on joint D; S is required; S is known; S and S are unknown, and are not required at the present time. S may be resolved at the point c in its line of action into S_y and S_x, its vertical and horizontal components, respectively. If the point e is taken as the center of moments, the moments of S, S_y, and S_x will all be zero. Then,

$$\Sigma M = S_y \times 2p - S \times p = 0;$$

whence
$$S_y = \frac{S}{2} = \frac{W}{2}$$

and
$$S = S_y \times \csc H = \frac{W}{2} \csc H, \text{ compression}$$

In other words, the vertical component of the stress in a short diagonal is equal to one-half the panel load at the top and at the foot of the short vertical. Then, *the maximum compression in a short diagonal is equal to one-half the sum of a dead and a live panel load multiplied by csc H; the minimum compression (except in eF) is equal to one-half of a dead panel load multiplied by csc H.* The minimum stress in eF will be discussed later in connection with counter eFG. In case part of the dead load is applied at the point D, this must be added to the panel load at d in getting the stress in cD.

78. Stresses in the Hip Verticals.—The stress in the hip vertical Cc may be found by considering the joint c as a free body [Fig. 34 (e)]. The forces acting at c are W, S, S, S, S, and S. The equation $\Sigma Y = \Sigma S \sin H = 0$ gives

$$\Sigma Y = S - S \sin H - S \sin H - W = 0;$$

whence

$$S = S \sin H + S \sin H + W, \text{ tension}$$

S_5 and S_6 are the stresses in the short diagonals Bc and cD; therefore, $S_5 \sin H$ and $S_6 \sin H$ are each equal to $\frac{W}{2}$, and

$$S_4 = \frac{W}{2} + \frac{W}{2} + W = 2\,W$$

It is thus seen that *the maximum tension in the hip vertical is equal to twice the sum of a dead and a live panel load; and that the minimum tension is equal to twice a dead panel load.*

79. Stresses in the Main Diagonals.—The stress in the lower half of a main diagonal may be found by considering the portion of the structure to the left of a plane cutting that member and two chord members. For example, for the stress in De, Fig. 34 (a), the truss may be cut by a plane at the section q. The vertical component of the stress in the diagonal is equal to the shear on the section. The stress in the end post is compression; in the other diagonals, it is tension. The maximum stress in the lower half of a main diagonal to the left of the center is equal to the maximum positive shear multiplied by csc H; the minimum stress is equal to the minimum shear multiplied by csc H, when the minimum shear is positive. When the minimum shear is negative, a counter is required, and the minimum stress in the main diagonal is zero.

The stress in the upper half of a main diagonal may be found by considering the forces acting on the portion of the structure to the left of a plane of section that cuts the upper half of that diagonal, a short diagonal, and two chord members. For example, for the stress in EF, Fig. 34 (a), the portion of the truss to the left of section r may be considered [see Fig. 34 (f)]. The equation $\Sigma\, Y = \Sigma\, S \sin H = 0$ gives

$$\Sigma\, Y = R_1 - 4\,W - S_{12} \sin H - S_{11} \sin H = 0;$$

whence

$$S_{11} \sin H = \text{shear on section } r - S_{12} \sin H$$

S_{12} is the stress in the short diagonal eF, and is equal to one-half the panel load at f multiplied by csc H. Then,

$$S_{12} \sin H = \frac{W}{2}$$

and

$$S_{11} = \left(\text{shear on section } r - \frac{W}{2}\right) \times \csc H$$

In other words, *the stress in the upper half of a main diagonal is equal to the algebraic sum of the shear in the panel in which the member is located and the vertical component of the stress in the short diagonal, multiplied by csc H.*

The maximum stress in BC obtains when the truss is fully loaded, and is compression; the vertical component of the stress is equal to the shear in the panel bc *plus* the vertical component of the stress in Bc. (The load at b increases the vertical component of the stress in BC by $\frac{11}{12} W - W + \frac{6}{12} W = \frac{5}{12} W$). The minimum stress obtains when there is no live load on the truss.

The maximum stress in CD is tension, and obtains when all the joints from d to b' are loaded with a live load. (The load at d increases the vertical component of the stress in CD by $\frac{9}{12} W - \frac{6}{12} W = \frac{3}{12} W$.) The minimum stress obtains when joints b and c are loaded with live load.

The maximum stress in EF is tension, and obtains when all the joints from f to b' are loaded with live load. (The load at f increases the vertical component of the stress in EF by $\frac{7}{12} W - \frac{6}{12} W = \frac{1}{12} W$.) The minimum stress obtains when all the joints from b to e are loaded with live load.

If the minimum combined shear in panel ef is positive and greater than a dead panel load, the shear in panel fg will also be positive, and the main diagonal gFE will be in action. The stress in EF is then equal to the shear in panel ef minus one-half of the dead load at f, multiplied by csc H. If the minimum shear in the panel ef is negative, or positive and less than the dead load at f, the shear in the panel fg for the same loading will be negative, and the member FG will be in action as a counter. Under this condition, the stress in the lower half of the main diagonal gF will be zero, and the members that are in action at joint F will be those shown in Fig. 34 (g). It will be seen that the stress in EF is thus equal to one-half the dead load at F multiplied by csc H; this is the minimum stress in EF, and is tension.

80. Stresses in the Counters.—The stress in eF is a minimum (maximum tension) when the live load extends

from b to e. If the combined shear in the panel ef is then negative, the stress in eF is tension and equal to the shear plus one-half the dead load at f, multiplied by csc H. If the combined shear in panel ef is then positive and less than one-half the dead load at f, the stress in eF is tension and equal to the difference between one-half the dead load at f and the shear in the panel ef, multiplied by csc H. If the shear in panel ef is positive and greater than one-half, but less than a full, panel dead load at f, the stress in eF is compression and equal to the shear in the panel ef minus one-half the dead load at f, multiplied by csc H.

The maximum tension in FG is equal to the maximum negative shear in the panel fg multiplied by csc H, and obtains when the live load extends from b to f. The minimum stress is zero.

81. Stresses in the Verticals.—The maximum stress in Gg is compression, and is equal to the vertical component of the maximum stress in FG plus the dead load at G, if any; the minimum stress is zero, and obtains when the counters FG and $F'G$ are out of action. The maximum stress in Ee is compression, and is equal to the vertical component of the maximum stress in EF; the minimum stress is equal to the vertical component of the minimum stress in EF.

82. Modified Baltimore Truss.—Fig. 35 shows a modified form of the Baltimore truss, in which the short

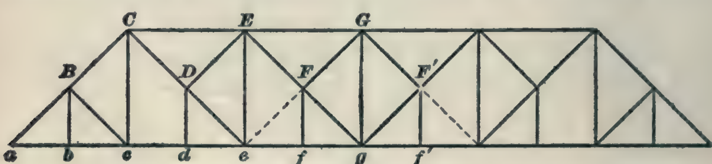


FIG. 35

diagonals (except Bc) are attached to the upper-chord joints, and are tension members. The general method of analysis is the same as for the truss already treated.

83. Deck Truss.—When used in a deck bridge, the Baltimore truss may be supported as shown in Fig. 36.

In this form, the subverticals bB, dD, etc. are compression members, and the short diagonals Bc, De, etc. are tension members. The general method of analysis is the same as for the through truss.

84. Distribution of Dead Load. If the dead load is assumed to be divided between the loaded and unloaded chords, two-thirds of a dead panel load may be taken at the joints of the loaded chord (b, c, d, etc.), one-third at the joints of the unloaded chord (C, E, etc.), and one-third at the intermediate joints (B, D, etc.), although the latter are not on the unloaded chord. In the preceding discussion, it was assumed for convenience in explanation that all the dead load was applied at the joints of the loaded chord. The actual stresses in the verticals will be slightly different if the dead load is distributed among the several joints; the stresses in the other members will remain the same.

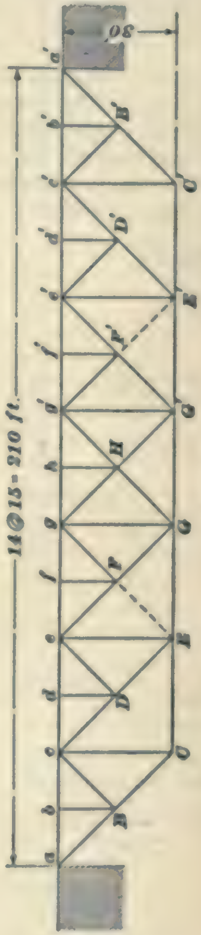


FIG. 36

EXAMPLE.—Let Fig. 36 be a fourteen-panel deck Baltimore truss having a span length of 210 feet and a height of 30 feet. If the dead load is 1,200 pounds, two-thirds of which is applied at the loaded chord, and the live load is 2,400 pounds per linear foot, what are the maximum and minimum stresses in all the members?

SOLUTION.—*Panel Loads and Reactions.* The dead panel load is

$$\frac{1,200}{2} \times 15 = 9,000 \text{ lb.}$$

of which 6,000 lb. is applied at each of the joints b, c, d, etc.; 3,000 lb. at B, D, F, etc.; and 3,000 lb. at C, E, G, etc. The dead-load reaction is equal to

$$\frac{9,000 \times 13}{2} = 58,500 \text{ lb.}$$

The live panel load is equal to

$$\frac{2{,}400}{2} \times 15 = 18{,}000 \text{ lb.}$$

and the reactions for a fully loaded truss are each

$$\frac{18{,}000 \times 13}{2} = 117{,}000 \text{ lb.}$$

Chord Stresses.—As the loads of 6,000 lb. at b, c, d, etc. are vertically over the loads of 3,000 lb. at B, C, D, etc., the moment of 6,000 lb. at an upper joint, plus the moment of 3,000 lb. at a lower joint, about any point will be the same as the moment of 9,000 lb. about the same point. The minimum stresses in the chords are the dead-load stresses; they are as follows:

Member	Dead-Load Stress, in Pounds
ab, bc	$\dfrac{58{,}500 \times 15}{15} = +\ 58{,}500$
cd, de	$\dfrac{[58{,}500 \times 4 - 9{,}000\,(3+2)] \times 15}{30} = +\ 94{,}500$
ef, fg	$\dfrac{[58{,}500 \times 6 - 9{,}000\,(5+4+3+2)] \times 15}{30} = +\ 112{,}500$
gh	$\dfrac{[58{,}500 \times 8 - 9{,}000\,(7+6+5+4+3+2)] \times 15}{30} = +\ 112{,}500$
CE	$\dfrac{(58{,}500 \times 2 - 9{,}000 \times 1) \times 15}{30} = -\ 54{,}000$
EG	$\dfrac{[58{,}500 \times 4 - 9{,}000\,(3+2+1)] \times 15}{30} = -\ 90{,}000$
GG'	$\dfrac{[58{,}500 \times 6 - 9{,}000\,(5+4+3+2+1)] \times 15}{30} = -\ 108{,}000$

As the total panel load is equal to 18,000 lb. live load plus 9,000 lb. dead load, or 27,000 lb., the maximum chord stresses are equal to the dead-load stresses multiplied by $\frac{27{,}000}{9{,}000}$, or 3. They are as follows:

Member	Maximum Stress, in Pounds
ab, bc	$+\ 58{,}500 \times 3 = +\ 175{,}500$
cd, de	$+\ 94{,}500 \times 3 = +\ 283{,}500$
ef, fg	$+\ 112{,}500 \times 3 = +\ 337{,}500$
gh	$+\ 112{,}500 \times 3 = +\ 337{,}500$
CE	$-\ 54{,}000 \times 3 = -\ 162{,}000$
EG	$-\ 90{,}000 \times 3 = -\ 270{,}000$
GG'	$-\ 108{,}000 \times 3 = -\ 324{,}000$

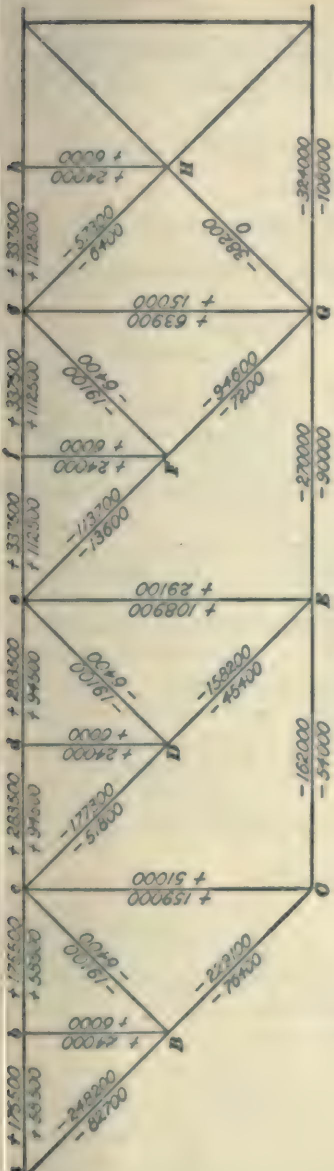


FIG. 37

Web Stresses.—The dead-load shears in the various panels are as given in the following table:

Panel	Dead Load Shear Pounds	Panel	Dead Load Shear Pounds
a b	+ 58,500	*e f*	+ 22,500
b c	+ 49,500	*f g*	+ 13,500
c d	+ 40,500	*g h*	+ 4,500
d e	+ 31,500		

The maximum positive and negative live-load shears are as follows:

Panel	Positive Live-Load Shear Pounds	Negative Live-Load Shear Pounds
a b	+ 117,000	
b c	+ 100,300	− 1,300
c d	+ 84,900	− 3,900
d e	+ 70,700	− 7,700
e f	+ 57,900	− 12,900
f g	+ 46,300	− 19,300
g h	+ 36,000	− 27,000

The maximum and minimum combined shears are as follows:

Panel	Combined Shear	
	Maximum	Minimum
a b	+ 175,500	+ 58,500
b c	+ 149,800	+ 48,200
c d	+ 125,400	+ 36,600
d e	+ 102,200	+ 23,800
e f	+ 80,400	+ 9,600
f g	+ 59,800	− 5,800
g h	+ 40,500	− 22,50.

From the figure, $\csc H = \frac{\sqrt{30^2 + 30^2}}{30} = 1.414$. The maximum stress in bB, dD, etc. is $18,000 + 6,000 = 24,000$ lb., compression. The minimum stress in bB, dD, etc. is 6,000 lb., compression. The maximum stress in Bc, De, and Fg is

$$\left(\frac{18,000 + 6,000 + 3,000}{2}\right) \times 1.414 = 19,100 \text{ lb., tension}$$

The minimum stress in Bc, De, and Fg is

$$\left(\frac{6,000 + 3,000}{2}\right) \times 1.414 = 6,400 \text{ lb., tension}$$

The maximum and minimum stresses (tension) in aB, cD, eF and gH are as folows:

Member	Stress, in Pounds	
	Maximum	Minimum
aB	$175,500 \times 1.414 = 248,200$	$58,500 \times 1.414 = 82,700$
cD	$125,400 \times 1.414 = 177,300$	$36,600 \times 1.414 = 51,800$
eF	$80,400 \times 1.414 = 113,700$	$9,600 \times 1.414 = 13,600$
gH	$40,500 \times 1.414 = 57,300$	$4,500 \times 1.414 = 6,400$

The maximum and minimum stresses (tension) in BC, DE, FG, and GH are as follows:

Member	Maximum Stress, in Pounds
BC	$(175,500 - 27,000 + 13,500) \times 1.414 = 229,100$
DE	$(125,400 - 27,000 + 13,500) \times 1.414 = 158,200$
FG	$(80,400 - 27,000 + 13,500) \times 1.414 = 94,600$
GH	$(22,500 + 4,500) \times 1.414 = 38,200$

Member	Minimum Stress, in Pounds
BC	$(58,500 - 9,000 + 4,500) \times 1.414 = 76,400$
DE	$(36,600 - 9,000 + 4,500) \times 1.414 = 45,400$
FG	$(9,600 - 9,000 + 4,500) \times 1.414 = 7,200$
GH	0

The maximum and minimum stresses (compression) in Cc, Ee, and Gg are as follows:

Member	Stress, in Pounds	
	Maximum	Minimum
Cc	$162,000 - 3,000 = 159,000$	$54,000 - 3,000 = 51,000$
Ee	$111,900 - 3,000 = 108,900$	$32,100 - 3,000 = 29,100$
Gg	$66,900 - 3,000 = 63,900$	$4,500 + 6,000 + 4,500 = 15,000$

The maximum and minimum stresses are given in Fig. 37, the former above and the latter below the lines.

EXAMPLE FOR PRACTICE

Let Fig. 34 (*a*) be a twelve-panel through Baltimore truss having a span length of 216 feet and a height of 36 feet. If the dead load is 1,200 pounds, of which two-thirds is applied at the loaded chord, and

the live load is 2,200 pounds, per linear foot of bridge, find the maximum and minimum combined stresses: (*a*) in CE, cd, and fg; (*b*) in Dd and Bc; (*c*) in BC, CD, and EF; (*d*) in eF, Ee, and Gg.

		Panel	Stress, in Pounds Maximum	Stress, in Pounds Minimum
Ans.	(*a*)	CE	+ 244,800	+ 86,400
		cd	− 168,300	− 59,400
		fg	− 260,100	− 91,800
	(*b*)	Dd	− 27,000	− 7,200
		Bc	+ 21,600	+ 7,600
	(*c*)	BC	+ 216,300	+ 76,400
		CD	− 136,800	− 38,800
		EF	− 66,600	− 7,600
	(*d*)	eF	+ 21,600	− 8,100
		Ee	+ 50,700	+ 9,000
		Gg	+ 22,900	+ 3,600

THE FINK AND THE BOLLMAN TRUSS

85. Fig. 38 shows the **Fink truss**, which has been used to some extent for bridge purposes in the past, and is at present employed in a modified form for roof trusses. The analysis of stresses in it should present no difficulty; the method of joints is best adapted to this case. The stress in each short vertical is evidently equal to the panel load at its upper joint. The vertical component of the stress in a short

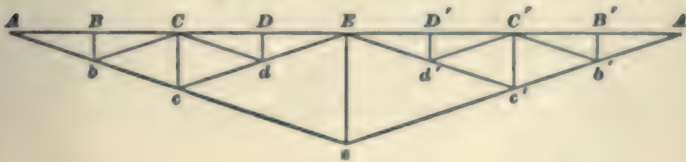


Fig. 38

diagonal, such as Cd, is equal to one-half the sum of the loads at the joints D and d. The stress in the vertical Cc is equal to the sum of the vertical components of the stresses in bC and Cd, and the load at C, etc. The maximum chord stresses obtain when there is a full load; under this condition the horizontal components in bC, dE, and $d'C'$ are equal, respectively, to those in Cd, Ed', and $C'b'$, and the compression in the top chord is constant from A to A' and equal to the horizontal component of the stress in Ab.

86. The **Bollman truss**, shown in Fig. 39 (*a*), has been used to some extent for bridge purposes in the past, but is now practically obsolete. The load that came on the lower joint of a vertical was carried directly to the ends of the top chord by the two diagonals, shown in full lines, that meet at the bottom of the vertical. The bottom chord and

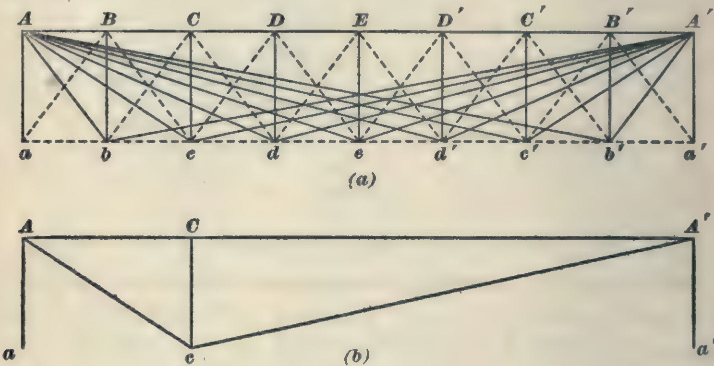


FIG. 39

the dotted diagonals are superfluous members, and were put in to stiffen the truss. As shown in Fig. 39 (*b*), the vertical components of the stresses in the main diagonals are equal, respectively, to the reactions due to the load that comes to the intersection of any pair. The stress in the top chord is compression, and is equal to the sum of the horizontal components of the stresses in all the main diagonals at one end of the truss.